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Publication Details
Department of Education
GPO Box 169
HOBART TAS 7000
web: www.education.tas.gov.au

Version 1
Published: February 2016

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ACARA Acknowledgements
Parts of this publication are based on Australian Curriculum, Assessment and Reporting Authority (ACARA) materials.
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Supporting school improvement and quality teaching

Our Learners First Strategy strengthens the quality of teaching and learning opportunities for all students in our system. In order to work effectively, well-developed teaching and learning programs need to be implemented in every school, supported by strong, instructional leadership.

We know that it is good teaching that makes the difference to our students. The rich resources that are provided in the Good Teaching series are successfully supporting teachers and school leaders to continue to build both collaborative practice and a whole school approach to school improvement K–12.

Building on the Good Teaching series and aligned to the Supporting Literacy and Numeracy Success booklet, a set of literacy and numeracy resources has been developed to give teachers in the early years through to Year 12 more support and confidence when planning for students’ literacy and numeracy needs across the curriculum. This particular resource focuses specifically on teaching numeracy 3–6.

Supporting professional learning

Our Learners First Strategy aims to develop successful, skilled and innovative Tasmanians. Its values include learning and excellence so that Tasmanians are engaged in positive, productive and supported learning experiences, and have high expectations and a strong commitment to the pursuit of excellence.

As with the other Good Teaching resources, this resource will be accompanied by a professional learning program through the Professional Learning Institute (PLI) available to all schools.

It forms part of the Good Teaching series of resources that also includes:

**Good Teaching: A Guide for Staff Discussion**

The purpose of this guide is to raise the debate across schools to gain a common understanding of what makes a good teacher. It is the foundation of the Good Teaching series.


**Good Teaching: Differentiated Classroom Practice – Learning for All**

It is recognised that some students require significant adjustments to their learning programs if they are to be optimally engaged and challenged. The process of making those adjustments is known as the differentiation of classroom learning. Differentiation is what is expected of good teachers. The focus of this resource is to describe what is meant by differentiation and to provide practical strategies and tools that can be used to create meaningful and engaging learning experiences for all students.

**Good Teaching: Curriculum Mapping and Planning – Planning for Learning**

Curriculum mapping and planning is a way of developing a systematic overview of what students need to learn. It provides an opportunity to evaluate current practice and fosters communication among teachers at all levels and across a range of subjects. This resource describes processes that schools and individual teachers can use to move from curriculum frameworks to classroom action. It provides guidance for planning directly from curriculum documents. Specific examples are provided for Australian Curriculum: English and Mathematics.


**Good Teaching: Quality Assessment Practices – Guiding Learning**

This resource supports schools in their school improvement agenda by describing processes that will guide leaders and teachers in the use of quality assessment practices. It supports schools in the choice of evidence-based strategies to meet the learning needs of all students. When used in conjunction with differentiated classroom practice, it supports teachers to adjust strategies to meet individual needs.


**Good Teaching: Inclusive Schools – Disability Focus**

This resource addresses key strategies in inclusive education through:

- improved teaching quality and support
- a robust national curriculum
- better support for students
- improved parent and community information and participation.


**Good Teaching: Inclusive Teaching for Students with Disability**

This resource follows on from the Good Teaching: Inclusive Schools – Disability Focus resource and has been developed for teachers who have not previously worked with students with disability.

It is a practical resource to develop teachers’ skills and confidence in this area and outlines the different areas of support available across the school and the Department in working with students with disability and their families.

Practical examples are provided using the following identifiers:

- Template
- Good Practice
- Video
- Tool
- Resources
- Conversation Starters

How the content is organised:

The booklet is divided into colour coded sections. Each section begins with key messages for Years 3–6 educators followed by conversation starters to initiate rich discussion in staff meetings or professional learning communities.

In each of the sections A-F there is a focus on specific links to the elements of the general capability of numeracy and the Australian Curriculum: Mathematics to support classroom teachers in knowing the key teaching focuses at their year level. These sections also provide practical ideas for teachers and suggestions for the types of activities, questions and materials they might use to support student learning of numeracy.

At the back of the booklet there are references and recommended resources (including assessment tools) to provide additional support to teachers and school leaders for a more thorough appreciation of the key messages.

This resource should also be used in conjunction with:

- Supporting Literacy and Numeracy Success which provides teachers with strategies for improving literacy and numeracy outcomes as they plan using curriculum documents.

- Respectful Schools Respectful Behaviour which highlights the importance of providing safe and supportive environments as a vital part of quality teaching and learning.

- Curriculum in Tasmanian Schools K–12 Policy

- Assessment and Reporting Policy

NAPLAN Toolkit

The NAPLAN Toolkit supports teachers with strategies for teaching key concepts in literacy and numeracy. http://naplan.education.tas.gov.au
BECOMING NUMERATE

Numeracy across the years of schooling

Students become numerate as they develop the knowledge, skills and dispositions to interpret and use mathematical skills and understandings purposefully across their years of schooling K–12, across all learning areas and in their daily lives. Students develop their knowledge, skills and confidence with numeracy as they connect and apply their understanding of mathematics to contexts within and beyond the classroom.

In the early years K–2, numeracy is developed through play and through informal yet intentional teaching in social contexts. Respectful relationships between schools and families are critical in the early years; the best numeracy outcomes are likely to be achieved when teachers work in partnership with students, parents, carers and the community and share responsibility for learning.

Numeracy is prioritised in the Australian Curriculum across Years 3–6. The curriculum continues to progress the development of specific mathematical skills and knowledge, and uses these skills in learning across the curriculum to both enrich the study of other learning areas and contribute to the development of broader and deeper numeracy skills.

In the 7–10 years, numeracy is a key driver of learning across all Australian Curriculum areas. Students use their understandings of numbers, patterns, measurement, spatial reasoning and data across different learning areas. Teachers support students to analyse a situation to identify the mathematical ideas involved, asking the question: ‘How can maths help here?’ Students use mathematical skills and understandings for increasingly specialised purposes and audiences in a range of contexts. In doing so, students become confident communicators, critical thinkers, and informed young people who understand the world around them.

Numeracy across the curriculum

Students become numerate as they engage with numeracy opportunities and experiences across the learning areas of the Australian Curriculum and in learning linked to the outcomes of the EYLF. Numeracy happens when students understand the role of mathematics and have the dispositions and capacities to use mathematical knowledge and skills purposefully. Examples of becoming numerate in learning across the curriculum can be found on the Australian Curriculum website. The following extract can be accessed from http://www.australiancurriculum.edu.au/generalcapabilities/overview/general-capabilities-in-the-learning-areas
In **English** students develop numeracy capability when they interpret, analyse and create texts involving quantitative and spatial information such as percentages and statistics, numbers, measurements and directions.

In **Mathematics** students develop numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. It is important that the Mathematics curriculum provides the opportunity to apply mathematical understanding and skills in context. A particularly important context for the application of Number and Algebra is financial mathematics. In Measurement and Geometry, there is an opportunity to apply understanding to design. The twenty-first century world is information driven, and through Statistics and Probability students can interpret data and make informed judgments about events involving chance.

In **Science** students develop numeracy capability when they collect both qualitative and quantitative data, which is analysed and represented in graphical forms and through learning data analysis skills, including identifying trends and patterns from numerical data and graphs.

**Humanities and Social Sciences (HASS)**

- In **History** students develop numeracy capability as they learn to use scaled timelines, including those involving negative and positive numbers, as well as calendars and dates to recall information on topics of historical significance and to illustrate the passing of time.

- In **Geography**, students develop numeracy capability as they investigate concepts of location and distance, spatial distributions and the organisation and management of space within places; in constructing and interpreting maps, students work with numerical concepts of grids, scale, distance, area and projections.

- In **Economics and Business** (from Year 5) students use numeracy to understand the principles of financial management, and to make informed financial and business decisions.

- In **Civics and Citizenship** (from Year 3) numeracy knowledge and skills are applied to analyse, interpret and present information in numerical and graphical form, including conducting surveys and representing findings in graphs and charts.

Across the **Arts** subjects, students use spatial reasoning to solve problems involving space, patterns, symmetry, 2D and 3D shapes; scale and proportion and measurement to explore length, area, volume, capacity, time, mass and angles.

In **Technologies** students cost and sequence when making products and managing projects. They use three-dimensional models, create accurate technical drawings, work with digital models and use computational thinking in decision-making processes when designing and creating best-fit solutions.

In **Health and Physical Education** students use calculation, estimation and measurement to collect and make sense of information related to nutrition, fitness, navigation in the outdoors or various skill performances. They use spatial reasoning in movement activities and in developing concepts and strategies for individual and team sports or recreational pursuits.

In **Languages** there are opportunities for learners to use the target language to develop skills in numeracy, including processes such as using and understanding patterns, order and relationships to reinforce concepts such as number, time or space in their own and in others’ cultural and linguistic systems.

In **Work Studies** (Years 9–10) students strengthen their numeracy skills by making direct connections between their mathematical learning and the nature of mathematics required in workplaces and enterprises. Students recognise that financial literacy is a requirement across enterprises and that numeracy helps them manage salaries and personal and workplace budgets and calculate personal and enterprise tax liabilities.
Wholesale school approaches to numeracy

Numeracy is regarded as one of the key dimensions of all learning. Increases in numeracy outcomes will only occur when there is a sustained whole school commitment to systematic curriculum delivery over a period of time. Improvement in numeracy achievement requires a whole school commitment to the following key aspects:

Organisation
A culture of collaboration empowers staff to work together on numeracy provision, discussing, reflecting, planning, setting goals, developing resources, analysing data and work samples, and sharing learning. An effective numeracy improvement strategy benefits from a numeracy leader and/or a numeracy team managing and leading:

- planning and review cycles
- collecting and analysing data
- target setting
- ensuring coherence and continuity across the years
- promoting formative assessment strategies to guide numeracy teaching
- building staff capacity and confidence
- enabling the sharing of effective numeracy practices across the school
- supporting the principal to develop whole-of-school interventions for students requiring differentiated numeracy support
- identifying resource needs and allocation
- convening and structuring year group numeracy planning meetings.

Planning
It is important for schools to allocate time for grade and section meetings so that teachers can plan how to integrate numeracy activities into learning experiences and plan units of work for the explicit teaching of the mathematical skills needed for numeracy. Teachers collaboratively decide on consistent approaches to numeracy practices and assessment, share and confirm numeracy language, and develop numeracy resources to support all learners.

Principal and the leadership team have a role in supporting teachers to understand the progressive development of numeracy across the years of schooling as identified in the Australian Curriculum Numeracy Learning Continuum and Australian Curriculum scope and sequence for mathematics.

Teacher planning includes differentiating the curriculum for students requiring additional numeracy support; including students who are highly able, gifted and talented and those who are not making expected progress. Refer to Good Teaching: Differentiated Classroom Practice and Supporting Literacy and Numeracy Success.

Teaching and learning
An inquiry approach to teaching and learning numeracy is recommended for all years of schooling. Thinking that is productive, purposeful and intentional is at the centre of effective learning (ACARA, Critical and Creative Thinking general capability). All students need to be explicitly taught skills to solve problems across the curriculum. As they become numerate, students use their mathematical understandings, skills, dispositions and strategies to solve problems they meet in a wide range of contexts.

At all levels of schooling students need to recognise which aspects of their mathematical knowledge are relevant to a particular situation, select and then apply those ideas and skills. This involves problem-solving skills and higher order thinking skills, for example, when making three-dimensional models, calculating length and area or planning a budget for a school social. The fundamental numeracy question is: ‘What maths will help here?’

Because numeracy involves context, it is usually described in words. For students to understand about how context changes the numeracy
demands of a situation we need to teach them to visualise and paraphrase; to see ‘in their mind’s eye’ what the words are saying and learn to talk about it in their own words – ‘If we need to plan our excursion, how many students will be going? How many seats are on the bus? How many lunch packs will we need? How will we transport lunches and keep them cool?’

Visualising and paraphrasing are the first steps to understanding and clarifying a situation and then solving a problem. Language, discussion and planned mathematical conversations support students in developing their numeracy.

The mathematics that underpins numerate behaviours is first learned as a body of knowledge in the primary years of school. By understanding a context and determining that ‘some mathematics will help here’, students then make some choices about what mathematics will help, and what strategies they will use to apply the mathematics chosen.

Students can then apply the mathematics and strategies chosen confidently and make a judgement about whether their solution makes sense in the context. If their solution does make sense then they gain confidence in their application of mathematics and are more likely to choose to use mathematics next time they identify it as being needed. Continued success will mean students become more and more numerate in many different contexts.

To be confident problem-solvers, students need to develop positive attitudes towards applying their mathematics knowledge and to be able to make strategic choices about which concepts and methods to apply in particular contexts. This requires careful planning for tasks which promote problem-solving, thinking and inquiry. Refer to Good Teaching: Differentiated Classroom Practice and Supporting Literacy and Numeracy Success.

Problem-solving is one of the mathematical proficiencies that should be used in teaching mathematics. However, the key numeracy question – will some maths help here? – is usually asked outside of, or prior to, mathematics lessons. Hence it is part of general problem-solving which can be used in every context – including other learning areas – and not just in the teaching and learning of mathematics.

The emphasis in Foundation to Year 6 schooling is on teaching the mathematics that students can draw on when confronted with a situation where some mathematics can help. Teachers in these years focus on supporting students to learn their mathematics in familiar contexts and to interpret the contexts by visualising and paraphrasing, as described earlier. These skills are part of a problem-solving strategy which all students should be taught as part of their numeracy development.

Problem-solving begins from the early years and progression is built into whole school planning for numeracy development. Problem-solving experiences also support students as they develop the general capability of Critical and Creative Thinking.

Effective numeracy teachers:

Effective Years 3–6 teachers recognise that they have particular roles in the development of students’ numeracy:

• Understanding how the ‘big ideas’ and concepts of mathematics develop through the curriculum and knowing the role they play in building these ideas in Years 3–6.

• Knowing where students are in their mathematical development in relation to these big ideas and concepts and how they use mathematics to demonstrate numeracy (this may require reference to the K–2 resource and targeted use of diagnostic assessment tools).

• Explicitly teaching the mathematical concepts needed for numeracy in ways that develop deep mathematical understandings in problem-solving contexts. This involves focus on the ‘new’ aspects of mathematics which build on from the K–2 learning, such as fractions, decimals, percentages and angles and the use of more precision in measurement.

• Continuing to build and strengthen the agreed whole school approaches to problem-solving which commenced in the K–2 years.

• Teaching students the increasingly complex mathematical vocabulary required to support their numeracy learning.
Whole school approaches to numeracy

• Planning learning tasks which challenge students to problem-solve, reason, demonstrate fluency and understanding; and developing assessment tasks which provide students with feedback on these proficiencies.

• Teaching mathematics as connected concepts, not methods or rules. This involves students using words to understand what they are doing and applying their mathematics across a range of contexts. Teaching mathematics skills alone is insufficient to develop the general capability of Numeracy.

In summary, to problem solve effectively in numeracy, students need to know how to:

1. Clarify the problem.
2. Choose the mathematics, tools, procedures and/or skills required.
3. Use and/or apply what has been chosen.
4. Interpret and check if the solution worked.
5. Communicate and talk about the steps they took to reach the solution.

Students need to be explicitly taught these steps if they are to be numerate. Knowing mathematics is essential, but it is not sufficient.

Support for teachers

Teachers have varying levels of experience and expertise in different aspects of numeracy education and they therefore require different levels of professional support.

• Having a numeracy leader or a numeracy team who, along with senior staff, can work with teachers ‘shoulder to shoulder’ as well as identifying their professional learning needs, underpins school improvement in this area of the curriculum.

• Professional learning is more effective when it is student-focused, data-informed, sustained and connected to school improvement plans, rather than in one-off sessions.

• Teachers need to be able to articulate both what they do and why they do it.

When these professional supports are in place, and when effective numeracy practices are shared across the school, whole school improvement is sustained.

Assessment

To ensure continuity of numeracy development, it is important to develop a consistent approach to assessment, as outlined in Good Teaching: Quality Assessment Practices.

• Whole school practices for collection and collation of data and reporting procedures support planning and tracking of student achievement.

• Teachers and leaders work together to investigate patterns of students’ strengths or underachievement and plan for interventions based on information from the data. Refer to Supporting Literacy and Numeracy Success.

• Assessment should lead to more effective teaching with teams developing a plan of action and selecting focus areas for improvement.

• Progress is monitored and teaching is adjusted accordingly.

• Year group teachers benefit from sharing formative numeracy assessment practices and planning for adjustments to teaching as a result of new understandings of learners.

• Success criteria should be shared with students, who increasingly take responsibility for addressing the criteria and assessing their own numeracy progress.

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1. Perso, T., 1999
NUMERACY 3–6

The numeracy learning environment

This resource identifies the key aspects of mathematics students in Years 3–6 need to understand in order to become numerate as they engage with numeracy across the curriculum and increasingly in their daily lives. It describes prerequisite features for numeracy learning as well as explicating the key elements of numeracy.

A classroom setting that encourages numeracy learning includes:

- Learning spaces designed to facilitate whole class, group, pair and individual work.
- Numeracy materials organised for independent learning e.g. counters, dice, measuring tools, calculators.
- Opportunities for experiential hands-on learning and classroom discussions focused on explaining and sharing learning.
- Various technologies e.g. individual mini-whiteboards, tablets, laptops, visualisers to project student work.
- Teacher/student made materials and posters (rather than commercial materials) that have a meaningful connection to the curriculum and are effective tools for teaching and learning.
- Frequently referenced materials e.g. mathematics dictionaries, number charts, number lines.
- Displays that are fresh, uncluttered and purposeful.
- Student work on display that shows thinking as well as their conclusions.

Planning and teaching for numeracy

In Years 3–6 teachers connect numeracy learning to the Numeracy general capability of the Australian Curriculum, underpinned by Years 3–6 of the Australian Curriculum: Mathematics. Effective planning and teaching emphasises backward design and the importance of clear links between learning goals and assessment tasks. Refer to Good Teaching: Curriculum Mapping and Planning. It includes:

- Assessing student understandings about concepts through oral questioning, watching students go about problem-solving and inviting input from parents about how their children deal with numeracy at home.
- Determining the mathematics that needs to be learnt by backward mapping from curriculum outcomes (both achievement standards and content descriptors from the Australian Curriculum: Mathematics and/or the expected indicators in the numeracy learning continuum).
- Determining the scaffolding and tasks needed to bridge between prior understandings and new numeracy and mathematical ideas.
- Determining contexts to teach the numeracy in familiar and engaging ways that promote discussion and focus on problem-solving.
- Facilitating inquiry learning, with planned opportunities for explicit focus on the mathematical ideas needed for numeracy.
- Teaching numeracy and mathematics in ways that promote understanding, support fluency, and demand reasoning and problem-solving.
- Determining the tasks required for students to demonstrate their understanding of numeracy.
- Generating data for diagnosing future learning and intervention needs, such as diagnostic assessment, benchmarking, outcomes assessment, success criteria (including rubrics), observation checklists, portfolios of student work and parent interviews (see Appendix).
Monitoring and assessment

In order to monitor the learning against what children are expected to learn, teachers need to refer to the intended learning in the Australian Curriculum.

Assessment must focus on assessing the intended learning rather than just what is currently being taught – we are assessing each student’s progress towards the desired goals. To assess the deep understandings of mathematics, as described above, teachers must ensure that they have taught the key ideas to all students using a high expectations approach and differentiation strategies as described in Good Teaching: Differentiated Classroom Practice. This may include the use of extending or enabling prompts for tasks/problems to broaden access to the learning.

If students don’t appear to be progressing it is important to have an objective look at your assessment tasks:

- Are the questions too hard?
- Do they assess the learning that you planned for?
- Are there words and phrases in the tasks that students may not be able to read and understand?
- Do your observations validate the learning, or are you seeing different things in student behaviours than you are seeing in written tasks?

Have a peer or mentor check the alignment between the tasks and the intended learning.

A good source of assessment questions for students in Years 3, 4 and 5 is some of the questions in past NAPLAN Numeracy test papers for Years 3 and 5. It is also important to expose Year 6 students to the types of questions they will meet in Year 7 through using questions from the Year 7 test. The questions generally align closely with the definition of numeracy and:

- provide no hints about the mathematics children should choose to use
- are for the most part written in contexts children understand before choosing the mathematics to use.

Past NAPLAN questions and associated teaching strategies can be accessed via the NAPLAN Toolkit and all teachers in Years 3–6 are encouraged to use this resource to inform their teaching. The IMPROVE website is an additional tool for formative assessment and also uses past NAPLAN questions.

A range of diagnostic tools which support teachers in making ‘fine grained’ assessments is listed in the Appendix.

Questions for reflection

For leaders:

- How is numeracy reflected in the School Improvement Plan?
- What organisational provisions are in place for developing whole school numeracy across the curriculum? e.g. instructional leadership, collaborative planning teams
- How are these responsibilities for numeracy distributed? e.g. is there a numeracy leader or a numeracy team?
- How successful is the collaborative planning for numeracy teaching and assessment in the school?
- What are some common numeracy assessments the school uses?
- How are the diverse numeracy needs of students catered for?
- How is numeracy communicated to parents and carers?
- How are numeracy interventions managed and resourced?
- How is data collection and analysis of numeracy managed in the school?

For teachers:

- What types of assessments provide you with a range of numeracy data about your students?
- What learning experiences assist your students to develop numeracy competence and confidence? What has worked well? What could be improved?
- In what ways is the learning environment conducive to numeracy learning/ What could be improved?
- How do you design learning tasks that allow students to develop and demonstrate numeracy understanding?
- How is numeracy learning differentiated to meet the needs of all students?
- How is numeracy evident in planning across the curriculum?
- Are there areas of numeracy education in relation to which you feel you might benefit from professional learning?
The organising elements for Numeracy are:

- Estimating and calculating with whole numbers
- Recognising and using patterns and relationships
- Using fractions, decimals, percentages, ratios and rates
- Using spatial reasoning
- Interpreting statistical information
- Using measurement.

Note the higher-order cognitive functions captured in the verbs used; words such as estimate and interpret are at a higher level than others used in the Australian Curriculum: Mathematics content descriptors such as recall, investigate, count and develop. These words reinforce for teachers the depth of learning of mathematics students need in order to choose to apply mathematics to contexts outside the mathematics classroom. Students who gain mathematics knowledge in a superficial way – lacking deep understandings – are unlikely to have the confidence to use mathematics when they can choose not to. They are unlikely to become fully numerate or to demonstrate sophisticated numerate behaviours.

The following section about teaching and assessing Numeracy key elements 3–6 is aimed at developing students’ deep understandings. It therefore describes appropriate pedagogies and assessment tasks/questions that develop and elicit higher-order responses from students.

### A. Estimating and calculating with whole numbers

**Key messages**

The positioning of the words ‘estimating’ with ‘calculating’ at the front of this element is a key message for teachers: for numerate behaviour, all calculation requires an element of estimation at the outset. In order to determine that a solution makes sense in context – an important part of numeracy – we need first to have a sense of the nature of the solution we are expecting.

Some teachers may believe that mental mathematics is less rigorous than written mathematics. Estimation in calculation relies on deep understandings about numbers and how they work, as well as deep understandings about mathematical operations. Neither of these understandings is necessarily needed for standard written mathematics algorithms. Teachers also need to know that teaching algorithms to students as a means of fostering fluency before deep understandings are established can potentially have harmful effects(ii). Students rarely deepen their understandings by only practising algorithms.

The key to teaching students how to estimate and calculate with whole numbers is to ensure that they deeply:

1. Understand what whole and decimal numbers are and that they can be represented in words, numerals and objects/drawings, and on number lines.
2. Understand operations and how they can be used to represent and solve problems.
3. Understand how to break down and partition whole and decimal numbers to help solve problems.

Students should have engaged with these concepts for whole numbers in Years K–2. In Years 3–6 they continue to consolidate these concepts using larger numbers, including decimals (decimal numbers are explained in Section C).

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(ii) Kamii and Dominick, 1998
At level 3 in the numeracy learning continuum (by the end of Year 4) students initially have these understandings with whole numbers up to four digits and then to five digits; and at level 4 (Years 5 and 6) with numbers larger than one million.

Links to the curriculum
Links to the Australian Curriculum: Mathematics are with the Number and Algebra strand, Year 3 to Year 6, and to the numeracy learning continuum levels 3 and 4: Estimating and calculating with whole numbers.

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<td>Investigate the conditions required for a number to be odd or even and identify odd and even numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognise, model, represent and order numbers to at least 10,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply place value to partition, rearrange and regroup numbers to at least 10,000 to assist calculations and solve problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investigate and use the properties of odd and even numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognise, represent and order numbers to at least tens of thousands</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Investigate number sequences including multiples of 3, 4, 6, 7, 8, 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Operate with numbers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognise and explain the connection between addition and subtraction</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recall multiplication facts of two, three, five and ten and related division facts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Represent and solve problems involving multiplication using efficient mental and written strategies and appropriate digital technologies</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recall multiplication facts up to 10 x 10 and related division facts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify and describe factors and multiples of whole numbers and use them to solve problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use estimation and rounding to check the reasonableness of answers to calculations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve problems involving multiplication of large numbers by one-or two-digit numbers using efficient mental, written strategies and appropriate digital technologies</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve problems involving division by one-digit number, including those that result in a remainder</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use efficient mental and written strategies and apply appropriate digital technologies to solve problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investigate everyday situations that use positive and negative whole numbers and zero</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Locate and represent these numbers on a number line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Relevant Australian Curriculum: Mathematics Content Descriptors

<table>
<thead>
<tr>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Money and financial mathematics</strong></td>
<td>Solve problems involving purchases and the calculation of change to the nearest five cents and without digital technologies</td>
<td>Create simple financial plans</td>
<td>Investigate and calculate percentage discounts of 10%, 20%, and 50%, on sale items, with and without digital technologies</td>
</tr>
</tbody>
</table>

**Extracts from the Australian Curriculum: Mathematics Achievement Standards**

- **Year 3**
  - Students recognise the connection between addition and subtraction and solve problems using efficient strategies for multiplication.
  - They count to and from 10,000. They recall addition and multiplication facts for single digit numbers.
  - They represent money values in various ways.

- **Year 4**
  - Students choose appropriate strategies for calculations involving multiplication and division.
  - They use the properties of odd and even numbers.
  - They recall multiplication facts to $10 \times 10$ and related division facts.
  - They solve simple purchasing problems.

- **Year 5**
  - Students solve simple problems involving the four operations using a range of strategies.
  - They check the reasonableness of answers using estimation and rounding.
  - They identify and describe factors and multiples.
  - They solve problems involving all four operations with whole numbers.

- **Year 6**
  - Students recognise the properties of prime, composite, square and triangular numbers.
  - They describe the use of integers in everyday contexts.
  - They solve problems involving all four operations with whole numbers.

### Australian Curriculum Numeracy Learning Continuum

<table>
<thead>
<tr>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Understand and use numbers in context</strong></td>
<td>Identify, describe and use numbers larger than one million</td>
</tr>
<tr>
<td>Model, represent, order and use numbers up to five digits</td>
<td></td>
</tr>
<tr>
<td><strong>Estimate and calculate</strong></td>
<td>Solve problems and check calculations using efficient mental and written strategies</td>
</tr>
<tr>
<td>Estimate a solution to a problem and then check the solution by recalling addition, subtraction, multiplication and division facts</td>
<td></td>
</tr>
<tr>
<td><strong>Use money</strong></td>
<td>Create simple financial plans, budgets and cost predictions</td>
</tr>
<tr>
<td>Estimate the change from simple purchases</td>
<td></td>
</tr>
</tbody>
</table>
Planning

Teachers should use the information in the ‘Links to the curriculum’ table to plan by backward mapping from the expected outcomes. Refer to Good Teaching: Curriculum Mapping and Planning. In particular, they need to understand precisely what each content descriptor means. Because the expectations are described in terms of what students should be able to know, do and understand, teachers need to ask: What do I need to do to enable them to do and know these things? This is the pedagogy question.

To plan effectively, teachers also need to have some understanding of what their students already know, understand and can do through pre-assessments and other ongoing formative assessments. Teachers should refer to Good Teaching: Quality Assessment Practices so they can build on this prior learning with the intended 3–6 learning. The K–2 booklet indicates what students might be expected to know, do and understand if they have had access to teaching that attends to the expected standards of the Australian Curriculum in these years. However, it is important not to assume that all 3–6 students have had this access. It will be useful to obtain a copy of the K–2 booklet, to be reminded of these earlier expectations and to be familiar with the teaching knowledge and strategies suggested for younger children in order to provide re-teaching or intervention if necessary.

The activities in the following ‘Putting it into practice’ section describe the sorts of learning opportunities teachers need to create to develop the required, stated learning, both for numeracy and the mathematics that underpins it. The activities are not intended to be a comprehensive list but describe the type of activities that are required – students might need more or less reinforcement depending on where they ‘are at’ in their current learning.

Putting it into practice

1. Model, represent, order and use numbers up to five digits (level 3) and numbers larger than one million (level 4)

In the Years K–2 students learned that numbers can be represented in different forms. This understanding – expected with numbers up to four digits, now extends to five digits. Students learned to model and represent numbers using words, numerals and objects or drawings; if they have these understandings it is not difficult to add an additional digit. In adding an additional digit however, in Years 3 and beyond students are now working with thousands instead of ones and this requires extending their understanding of place value.

Students need to have been taught place value through focusing on the first ‘family’ in the number system – the relationship between ones, tens and hundreds. This family is called the Ones family. Teachers can teach this using plastic straws. Students use one straw to represent ‘one’. They then count ten straws and put an elastic band around them. By doing this they can see that there are ten individual straws in their one bundle of ten – the only thing that is changed is that the straws are now in a group of ten; ten of these is one of these. (The benefit of using straws rather than pop sticks is that when you introduce tenths you can simply cut a straw into ten equivalent sections and students can see that each position to the right of the decimal point is ten times smaller).

<table>
<thead>
<tr>
<th>‘Ones family’</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
</tr>
</tbody>
</table>

Estimating and calculating with whole numbers
Some activities for place value learning

For those students who don’t as yet understand the relationship between the ones, tens and hundreds, or for revision prior to adding the thousands places, there is a range of activities to promote depth of understanding about place value.

When many of us went to school we used to say ‘hundreds, tens and units’. Now of course they are hundreds, tens and ones, mainly because we have ‘units of measurement’ such as centimetres and kilometres and this can be confusing for us and our students.

• Have students work in pairs to count the number of pop sticks in a basket. They bundle their pop sticks in groups of ten and record them using a H T O chart. They compare their final number with other students and check that the total is right by counting groups of tens and single ones. To make the numbers larger have them add the total number of pop sticks from each pair in the class to find how many there are in the class. Ask them to suggest strategies and ways to organise their data, applying prior learning in: ‘Interpreting statistical information’.

• Write a three-digit number on the board and ask students to make the number using their straws in groups and singles. Have them repeat this strategy in small groups, one writing the number, the others making the number shown.

• Write a four-digit number on the board and show students some straws in groups and singles. Ask: How many more straws do I need to make the number on the board? Can I group them to make them easier to count?

Students need to be explicitly taught that in each multi-digit number e.g. 35, 237, 108, etc. each numeral has a face value and a place value. The face value tells them the value of the face or ‘how many’ and the place value tells them the value of the place i.e. tens, hundreds or ones.

This understanding will help a great deal when students are learning about the Thousands family. In many classrooms students are taught that the next place in a number after the hundreds place in a multi-digit number is the thousands place i.e. for example, 3 576 can be represented as:

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>thousands</td>
<td>hundreds</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Unfortunately, this is not correct as this position represents the one thousands. This learning may result in many misunderstandings for students and it is important to avoid reinforcing this misconception. Alternatively, when students have gained deep understanding about how the digits in the ‘Ones family’ relate to each other – through bundling and re-grouping – teachers need to introduce the entire next family: the Thousands.
This is another pattern in the way our numbers work; the relationship between hundreds, tens and ones is merely repeated but with each ‘thousand’ representing ‘one thousand’.

<table>
<thead>
<tr>
<th>Millions family</th>
<th>Thousands family</th>
<th>Ones family</th>
</tr>
</thead>
<tbody>
<tr>
<td>hundred millions</td>
<td>ten millions</td>
<td>one millions</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

In this number there are 576 ones and 3 one thousands. Altogether there are ‘three thousand, five hundred and seventy-six’.

As well as the visual pattern, shown above, this is consolidated through an oral pattern where students, hearing a number spoken, learn to listen for the word ‘thousand’ when deciding how to write the number. They hear for example, two thousand, three hundred and nine and know that the word ‘thousand’ comes at the ends of the thousands family; there are two thousands, and the remainder are ones. They can write 2 309 and can check it as they read it back as two thousand three hundred and nine.

If they deeply understand this relationship there is no reason why you would not teach your students the entire Thousands family at once rather than building it up digit by digit (i.e. one thousands, ten thousands and then hundred thousands).

As indicated above, in teaching about place and face value in the early years we would make sure students are not taught that ‘zero’ is nothing. It is something since the word ‘zero’ or the symbol ‘0’ tells us there are no ones, or one thousands, or tens, for example.

**Some further activities**

- Write a multi-digit number on the board e.g. 349 625 and ask questions that don’t require any regrouping, such as:
  - What will my number look like if I subtract three hundred?
  - What will my number look like if I add three hundred thousand?
  - What will my number look like if I add twenty?

- Have students work in pairs; one writes a multi-digit number and the other has to read it aloud. The student who writes it down needs to listen and then tell their classmate if they are wrong and read it as it should be said.

- For homework have students write down the odometer reading in their parent’s/carer’s car and write down its value in words.

- Make up a hypothetical student’s maths task and have students work together in pairs to mark it. The task should contain many common mistakes, such as:

<table>
<thead>
<tr>
<th>Number in digits</th>
<th>Number in words</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000016</td>
<td>One million and sixteen</td>
</tr>
<tr>
<td>30005</td>
<td>Three hundred thousand and five</td>
</tr>
</tbody>
</table>

Students have learned in K–2 that all numbers can be represented in numerals, words, as objects/drawings, and on a number line. Using straws (using one straw as one ‘one’) is helpful for teaching that each place in a number is ten times smaller (or ten times greater) than the numeral to the left (or right). However, they shouldn’t need to learn this as a rule if they
deeply understand place value. They learn this by knowing the place value of each digit in a number.

It is difficult to represent numbers beyond one thousand in straws or objects. Bundling straws into groups of ten and combining these to make groups of one hundred helps students see the growth. However, bundling ten bundles of one hundred straws might be the practical limit in a classroom. That is why it is so essential to ensure that students learn this relationship deeply before larger numbers are introduced. Ideally, students should be able to work abstractly with numbers beyond 1 000.

As numbers increase in magnitude, unmarked number lines should continue to be used to develop deep understandings about numbers. The concept of equidistance between numbers is important. Keep on stressing this by having students draw their own number lines that go forwards or backwards from a given number.

For example, on the number line below, two numbers, 120 and 206 are marked but that is all. Have students work in groups or pairs to estimate the missing number and to justify or explain why they think that is the number.

![Number Line with Numbers 120 and 206](image)

An unmarked number line can replace the need for many written algorithms, particularly when adding and subtracting(iii).

2. Teaching students to use operations to represent and solve problems, estimating first and then checking their estimate against the solution

Before coming to school young children learned phrases such as ‘get one more’ and ‘take some away’ that provided the basis for understanding the operations of addition and subtraction. Teachers built on that language in K–2 using problems such as: How many will you have if you two put yours together? If Jane has five and Fred has three how many have they got altogether?

Stories were also used in the early years as a great source of addition and subtraction problems. Children heard and used words such as ‘altogether’, ‘total’, ‘take away’, ‘more than’, ‘difference’ and other common language that, in context, might mean add and subtract.

It is important that students learn these concepts in real or make-believe contexts before being introduced to the mathematical – and often abstract – terminology.

Teachers in the Years 3–6 should continue to focus on important common language used in addition and subtraction contexts. Use phrases and questions such as: How many more than? What is the difference? How many will be left over? What is the total? How many altogether? How many remain? Using them in both written and oral work helps to create common and shared whole school approaches, which is essential for promoting consistent numeracy development.

Symbols representing addition and subtraction are introduced with reference to common words such as total, altogether, difference, more than, since these are words that explain and describe common usage of the mathematical concepts. Students need to be explicitly taught the connections and how to translate between one representation and another i.e. words to symbols as indicated in the table below:

<table>
<thead>
<tr>
<th>Concept</th>
<th>Phrases that are used</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>How many altogether?</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>What is the total?</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Combine</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>What is the sum?</td>
<td>+</td>
</tr>
<tr>
<td>Subtraction</td>
<td>How many more than?</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>How many are left?</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>How many are left over?</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>How many will remain?</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>What is the difference between?</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>How many went away?</td>
<td>-</td>
</tr>
</tbody>
</table>

These concepts and phrases continue to be important regardless of the age of students; what is changing is the magnitude of numbers used as students’ facility and knowledge of number...
values increases. Knowledge of operations is best taught and assessed in contexts which require students to reason about the words and what they mean. For example:

- If Shelley has thirty-five more books than Jill who has twenty-nine books, how many does Shelley have?
- If Sam has twice as many comics as Jim, how many might Sam and Jim have each?

Students can be given a number of either similar or dissimilar objects and, working in groups, asked to write ten problems about combining and/or separating their objects. Their problems should be written in words and as number sentences using symbols. They can solve their problems and use an ‘=’ symbol to indicate that they have added or subtracted to find their solution. It is important that students are explicitly taught that the symbol representing the words ‘is equal to’ (i.e. ‘=’) is not an operator symbol but it describes the relationship between the numbers on either side. This idea of balance is a vital underpinning for later work with algebra.

Breaking down and partitioning whole numbers to help solve problems

The K–2 booklet explained that partitioning is about breaking numbers down into parts and also building them back up (iv). Knowing basic number combinations and facts is essential for mathematical fluency.

In that phase of schooling, students were given activities to help them understand (through seeing) that any number can be thought of as a sum or difference of other numbers. By visualising ten as four and six for example, their understanding that the sum of the parts is always equal to the whole, and that the whole subtract one of the parts is always equal to the other part, deepen’s the understanding necessary for addition, subtraction and the relationship between the two operations (i.e. that addition undoes – or is the inverse – subtraction, and vice versa). This can be generalised as ‘If $a = b + c$ (where $a$, $b$ and $c$ represent numbers) then $a – c = b$ and $a – b = c’$. This is true no matter what the size of the numbers, and $a$, $b$, and $c$, can represent multi-digit numbers not merely single-digit numbers; an important understand for algebra.

From Years 3–6, students can use this part-part-whole thinking and understanding when working with larger numbers. They know, for example, that $25652 = 25000 + 652$ and that $25652 – 652 = 25000$ and $25652 – 25000 = 652$. This thinking is particularly useful as the numbers students are working with become larger.

For example, by thinking of two hundred as 163 and 37, children can see that the sum of the parts (37 + 163) is the same as the whole, 200. They can also understand that 200 – 37 = 163 and that 200 – 163 = 37. So, if they start with 163 and add 37 and get 200, then to get back to 163 they subtract 37 away from their answer.

iv. Siemon, Beswick, Brady, Clark, Faragher, Warren, 2011
Additive thinking with partitioning is very useful for mental computation, especially if you deeply understand place value. For example, for adding 256 to any number students simply need to think of it as adding two hundred, adding another fifty, and then adding six. Similarly, for subtracting 143 from any number they simply think of it as subtracting one hundred, subtracting another forty, and then subtracting three. They can do this in their heads or use an unmarked number line (referred to earlier). For example,

<table>
<thead>
<tr>
<th>Think</th>
<th>On a number line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add 140 and 120</td>
<td>Start at 140, add one hundred, add twenty</td>
</tr>
<tr>
<td>Subtract 170 from 561</td>
<td>Start at 561, take two hundred, add 30</td>
</tr>
</tbody>
</table>

Whilst a number line can help some students perform addition and subtraction, a student who is a strong additive thinker may not need to use one; their understanding of place value is sufficient. For example, in adding 3 802 to 230 they merely think ‘three thousand eight hundred and two add two hundred is 4 002, add thirty is 4 032’. Students should practise doing calculations mentally from left to right, rather than right to left. For example, when adding 456 + 342 they should think ‘four hundred plus three hundred is 700, fifty plus forty is ninety, so that’s seven hundred and ninety, and six plus two is eight so that’s seven hundred and ninety eight’. While they are using this mental strategy they might write down 700, 90 and 8 as they go to help them recall the answers to the steps; this is a written strategy and quite appropriate. With practice students will be able to group and ungroup numbers mentally. They might consider 342 + 471 for example, and know that whilst 300 + 400 is 700, 40 + 70 will give them more than 100 so by looking across the entire number they will know to write down 800 at the start to compensate for this; or they might write down seven initially and then cross it out and write eight as they move across the number to the right, finishing with 813. These skills are essential for numerate people and should be encouraged, modelled and practised by teachers talking out aloud as they adjust their thinking and calculation strategies.

A note about rounding
Rounding is a needed strategy for estimation. In fact, the rule for rounding (all numbers ending in 5, 6, 7, 8, 9, are rounded upwards to the nearest 10; others are rounded downwards) is mostly used for estimation purposes only. All other rounding is mostly done according to the context in which it occurs.

Teaching mental strategies
From additive to multiplicative thinking
By Year 3, students should see and understand that counting ‘lots of’ or ‘groups of’ four is the same as repeatedly adding four, and that this repeated addition is multiplication. However, if students continue to only think of multiplication as repeated addition, using addition and counting strategies to determine ‘lots of’, they will not deeply understand multiplication. This often leads students to rely on algorithmic procedures for multiplication rather than using mental multiplication strategies and conceptual understanding.

In many ways, making the jump from addition to multiplication can be very challenging for students since they can’t necessarily ‘see’ or ‘touch’ what is happening. When young children

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add they can physically move objects together to create the result of 'adding'; when they take away they can physically move some objects away to create the result of 'taking'. However, when multiplying two numbers such as 3 and 2, (3 x 2, or 3 groups of 2) the '3' refers to the number of groups of 2. This is quite abstract and is why it is helpful to use the term 'groups of' consistently when teaching the concept of multiplication. Don’t just start using times or multiplied by; using ‘groups of’ is a necessary scaffold for teaching multiplication (vi).

Continuous practice with the commutativity of multiplication i.e. knowing that two numbers can be multiplied by each other in any order without changing the result, can also assist students with their calculations.

120 = 60 x 2 and 120 = 2 x 60; ‘one hundred and twenty is sixty groups of two and one hundred and twenty is also two groups of sixty’.

120 = 30 x 4 and 120 = 4 x 30; ‘one hundred and twenty is thirty groups of four and one hundred and twenty is also four groups of thirty’.

Continue to use the words ‘groups of’ until you are sure students are connecting these words with the ‘X’ symbol. Ask them to tell you what different groups can be made from e.g. 84, 140, 96, 225, different things.

From repeated subtraction to division

In the same way that multiplication is repeated addition, so division is repeated subtraction. Understanding of division as a concept begins with sharing. Students learned in the early years that when they divide one number by another they are breaking a quantity into equal groups or parts.

How students read and visualise a word problem will affect how they write it symbolically. For example, if the problem is: ‘I shared 20 apples equally among 5 people; how many apples will they each get?’ the symbolic representation will be the same as for: ‘I divided 20 apples into groups of five, and gave one group of the apples to each person. How many people were there?’ Both these situations can be represented by $20 \div 5$, even though the situations are different.

Unlike with print literacy and other symbolic representations students have encountered in their learning so far, division is not understood left to right. Students need to learn, in words, that division is understood right to left. For example, $12 \div 4$ is read as: ‘12 divided by 4’ but is understood as: ‘how many groups of 4 are there in 12?’

Students can also be asked to complete tables such as the following by working in pairs and groups, or to develop similar tables with gaps (in any of the three columns) for their peers to complete.

<table>
<thead>
<tr>
<th>Problem in words</th>
<th>Written as</th>
<th>Read as</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many threes in 156?</td>
<td>$156 \div 3$</td>
<td>‘One hundred and fifty-six divided by three’</td>
</tr>
<tr>
<td>How many groups of 15 pencils are there in 90 pencils?</td>
<td>$90 \div 15$</td>
<td>‘Ninety divided by fifteen’</td>
</tr>
<tr>
<td>If I have 160 CDs and give ten people the same number each, how many will they each get?</td>
<td>$160 \div 10$</td>
<td>‘One hundred and sixty divided by ten’</td>
</tr>
</tbody>
</table>

Teachers should give students a range of these problems represented in words and get students to write the symbols and words showing the operation they are performing.

Calculating

When confronted with a calculation that needs to be carried out, students should use their ‘in-built calculator’ i.e. their brain first. Almost all single – and double-digit calculations can be done mentally using a mental strategy (viii). If the numbers are too big to deal with mentally, then students face the choice of a written strategy or a computational tool strategy. The choice of strategy will depend on how exact the answer has to be. That is, an estimate using a mental strategy is adequate where an approximate answer is sufficient; a written calculation is necessary where an exact answer is required.

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vi. As with previous comments about ‘groups of’ it is important that children see these groups as countable units, whether they be one three, two threes, three threes, and so on or one group of three, two groups of three, three groups of three; they can drop the ‘groups of’ once they have the understanding.
Students need to be explicitly taught mental strategies. McIntosh (2004)(vii) suggests four generic strategies:

1. **My method** (students describe orally the strategy they used and how successful it was; this allows them to hear the different strategies used by others).

2. **How else?** (students devise a range of calculator strategies which result in the same answer; this allows them to do mental calculations at their own level of confidence).

3. **How will I calculate?** (students choose a suitable calculation method and give reasons for their choice; these can be mental, written and on a calculator).

4. **What’s related?** (students recognise calculations that are related to the one they might be doing e.g. $30 + 40 = 70$ is related to $300 + 400 = 700$ and $31 + 39 = 70$).

Strategies for teaching these and other mental computational skills can be found in McIntosh (2004)(vii).

When numbers are larger than those in the examples, students should still be able to estimate without using written methods or technological tools. If they can’t, this would suggest that they don’t deeply understand the numbers (and place value in particular) or that they don’t deeply understand the operations. In either case your intervention would be to support these individual students by re-teaching these two concepts rather than resorting to teaching standard algorithms.

For example, in calculating $351 + 28$ they might estimate first by thinking:

- Three hundred and fifty add twenty-eight is 378, add 1 is 379

In calculating the exact answer they might think:

- Thirty-five tens and two tens is thirty-seven tens, one and eight is nine
- Thirty-seven tens and nine ones is 379

If they need to support this thinking by writing things down as they go, they might write:

- $350 + 20 = 370$
- $1 + 8 = 9$
- $9 + 370 = 379$
This written support is entirely appropriate. Similarly for multiplication, in estimating $34 \times 207$, they might think:
- Thirty lots of 200 is six and three zeros
- 6 000; my answer will be a bit more than that because I’ve rounded both numbers down, by ignoring the four and the seven
In calculating the exact answer they might use a calculator or think and write:
- 30 lots of 200 = 6 000
- Four lots of seven is 28
- Answer is 6 000 + 28 = 6 028
For dividing quantities students would still estimate based on their understanding of place value and operations.
For example, in dividing 562 by 18 they might estimate first, thinking:
- Five hundred and sixty divided by about 20 is (five in every hundred which is 25) and three more is 28
- If they are expecting a number of this size they would then likely use a calculator to obtain the correct answer and check its reasonableness by comparing their calculated answer with their estimated answer of 28
Students should be encouraged to develop their own informal strategies; teachers need to model the ‘talking aloud their thinking and jotting down numbers and words as they go’ so that students see that there are many possible and valid methods.
Teachers could give students some calculations to do and have them work in pairs or groups to get the solutions. Explain that they might all use different ways of doing it and that’s okay. Have them share their way with the class separately later, talking aloud and jotting numbers and symbols on the board as they go and explaining their thinking about why it is right.
These informal strategies do not preclude standard algorithms. However, students need to recognise that standard algorithms are merely one way to do calculation. They should be encouraged to think about the problem they are working on and make a decision about which method best suits that context. Teachers can support this thinking by modelling different methods and encouraging students to use and different methods and justify why they selected particular methods for different problems.
Solving problems and checking calculations
By far the biggest challenge that students face in solving mathematical problems is writing a number sentence to represent the calculation required. Students need first to clarify the problem and describe what is happening in their own words. They can then write number sentences to represent the calculations they undertake.
Students need to practise writing number sentences by working in small groups or pairs. By doing this they have the advantage of hearing a range of different approaches from other students; this can create cognitive conflict that challenges their previous thinking. Students who have to defend their decision-making have to use appropriate language and need to understand what is happening. Teachers can model this behaviour to students. They can play at being ‘devil’s advocate’ to prolong the arguments and thus impel more and more children to come to an opinion and to hear that it is okay not to know the answer immediately. By thinking through a problem and engaging in struggle, students learn and develop ‘growth mind-sets’.
You might develop a table such as the following initially with students:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Paraphrased and/or visualised</th>
<th>Mathematics required in words</th>
<th>Symbolic representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>There were 25 673 people at a soccer match. One half of them supported the home side. About how many people supported the away team?</td>
<td>25 673 people at a soccer match. Roughly half are the away team, half are the home team. How many support the away team (estimate is enough)?</td>
<td>Find about half of 25 673</td>
<td>25 600 ÷ 2 =</td>
</tr>
<tr>
<td>There were some apples on a plate. Leena took six of them and then took two more. There were eight apples left. How many apples were on the plate at the start?</td>
<td>Some apples take six and then take two equals eight apples. How many apples did we start with?</td>
<td>Some take six take 2 equals 8. Start at 8, add 2 then add six will tell you how many we started with.</td>
<td>8 + 2 + 6 =</td>
</tr>
<tr>
<td>Eight people share a prize of $7 450. They keep $250 each and give the rest away to a charity. How much do they give to charity altogether?</td>
<td>Eight people have $250 each and give the remainder up to $7 450 away.</td>
<td>Work out what 8 lots of 250 is and then take the answer away from 7 450.</td>
<td>1. 8 X 250 = something 2. 7 450 – something = ?</td>
</tr>
<tr>
<td>Two students, Freda and Saul, get some pocket money each week. Freda is older so she gets $8 and Saul gets $5. After 9 weeks, Freda will have received more money altogether than Saul. How much more?</td>
<td>Both students get pocket money each week but Freda gets more than Saul because she’s older. After nine weeks how much more has Freda got than Saul?</td>
<td>Freda gets 9 lots of $8 Saul gets 9 lots of $5. Freda gets more. Take Saul’s total from Freda’s total.</td>
<td>1. 9 X $8 = $72 2. 9 X $5 = $45 3. $72 – $45 = ?</td>
</tr>
</tbody>
</table>
We must teach students these strategies if they are to successfully solve word problems. They need to read the words, comprehend their meaning and demonstrate that by visualising, drawing, paraphrasing. They then need to draw out the maths by breaking the questions into steps and writing a number sentence for each.

Money
In level 3 of the numeracy learning continuum it is important to note that the vowel used (i.e. the action required) is estimate. Students at this level are not required to make exact calculations in relation to money until the following year. This makes sense since in Year 4 they are just learning about decimal fractions and place value to tenths and hundredths.

Students who have achieved level 3 understand money well enough to estimate their change from a simple purchase. So, if they hand over $5 for their lunch they can add up the costs of what they bought (no more than one or two items) and say roughly how much they are expecting in change. They might say: I have two dollars and I'm going to buy an ice cream for 50 cents. That means I'll get one dollar fifty change or: A pie is $3.50 and with a drink that makes about $4.50. So that means I should get about fifty cents change.

The links between money and decimal fractions can be a complex idea for students and care must be taken in making the links explicit rather than merely assuming students can make the connections themselves. It is important that students are shown models other than money to support them in making sense of decimals; our currency does not go beyond two decimal places and is not thought of as a whole number and parts of a whole number. Generally we think of $2.50 as two dollars and 50 cents not two dollars and 50 hundredths.

For students to learn to estimate their change they need many opportunities to use and hear the words of estimating change. They need to hear teachers say things like: Forty five cents and thirty two cents is about eighty or ninety cents; one dollar forty and three dollars fifty is about five dollars; if I have ten dollars and spend eight dollars twenty I'll have close to (or just under) two dollars left.

Some activities for learning about money
Creating these situations in a classroom can often be very contrived. The usual 'classroom shop' can help. Have students bring along empty packets, boxes and other items and put exact prices on them in large print so they can see these numbers clearly. Do not round or make numbers simple to facilitate calculations. Remember that you want students to estimate their change and totals, not calculate. Estimating requires knowledge of numbers and number facts but not accurate calculation.

Students can ‘go shopping’ in pairs putting their chosen purchases (no more than two or three) in a basket, estimating the total as they go. Modelling this you might say: I have a jar of jam which is about one dollar and a bottle of shampoo which is close to four dollars. That’s about five dollars. Students can see the price on the jam is 95c and the price on the shampoo is $3.70 and yet hear you rounding the prices so the numbers are easier to work with.

Another strategy is to have students bring their junk mail to school and go through a catalogue circling two or three things they want to buy. Ask: Will you have enough if I give you five dollars? What about ten dollars? Roughly how much change will you have if I give you ten dollars? How much change will you get, roughly, if I give you fifteen dollars? Students may also visit an online supermarket site to do the same activity.

Level 4 of the numeracy learning continuum indicates that students should be able to: Create simple financial plans, budgets and cost predictions. This is a practical application of their learning about money and very appropriate for inclusion in the numeracy learning continuum.
Teachers need to make this as relevant to students’ lives as possible. Ask students if they receive any amounts of money regularly – weekly, monthly at birthdays, etc. Initial activities can be as simple as having students record their monthly amount in a spreadsheet and use it to calculate the amount they will have by the end of the year. This might look like:

<table>
<thead>
<tr>
<th>Month</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>August</th>
<th>September</th>
<th>October</th>
<th>November</th>
<th>December</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollars</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>160</td>
</tr>
</tbody>
</table>

You might teach them to use the $\sum$ (sum) function in Excel, and add up the total together.

Have students compare their monthly receipts with their yearly totals. Encourage them to think about scenarios like:

- If your parents gave you an additional $1 each month, what difference will this make to your monthly total?
- If your parents cut back your allowance by $5 a month, what difference would that make to your total?
- If you were told you had to pay $2 per week towards your phone plan (or pet food), what difference would that make to your total?
- If your allowance dropped by half the amount every week, what total would you get for the year?
- If your best friend received $2 per week less than you, would you have more or less than her/him in three months? How much less? In eight months? How much less?

Undertake a brainstorm with students about things they might be required to buy from their pocket money and how often those things occur. Examples might include:

1. Contributing to their phone payments.
2. Buying some of their clothing.
3. Paying for pet food.
4. Buying their own snacks at sports games.

Have students draw up a simple budget that includes these amounts. Show them that if they don’t know exact amounts, estimation is usually sufficient when predicting how much they will spend. If they need to keep an accurate budget so that they can plan for next year then estimation will not be sufficient.

Give them a (hypothetical) list of costs that you have to budget for each month and show them how to organise these in a table or spreadsheet. Have them calculate the yearly budget after discussing how that would be done.

Give students a hypothetical budget for themselves such as:

<table>
<thead>
<tr>
<th>Month</th>
<th>Activity</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>Sister’s birthday present</td>
<td>$10</td>
</tr>
<tr>
<td>January</td>
<td>Swimming pool entry fees</td>
<td>$12</td>
</tr>
<tr>
<td>May</td>
<td>Pay annual basketball fees</td>
<td>$20</td>
</tr>
<tr>
<td>July</td>
<td>Pay phone/data fees</td>
<td>$5/ month</td>
</tr>
<tr>
<td>April</td>
<td>Buy my friend a birthday present</td>
<td>$15</td>
</tr>
<tr>
<td>September</td>
<td>Buy Dad a Fathers’ Day present</td>
<td>$10</td>
</tr>
<tr>
<td>December</td>
<td>Buy Mum, Dad, brothers presents for Christmas</td>
<td>$50</td>
</tr>
</tbody>
</table>

Activities relating to this hypothetical budget might include:

- Calculate monthly payments
- Compare this with monthly receipts; are there any months when you won’t receive enough to cover your payments?
- What is the average payment per month?
- How does your total yearly allowance compare with your total yearly costs?
Other activities that students might find engaging and relevant include:

- Budgeting to keep a pet
- Budgeting for a party
- Budgeting to pay for iTunes or games
- Budgeting for clothes (to convince their parents they need more pocket money).

Monitoring and assessment

In order to monitor the learning against what children are expected to learn, teachers need to refer to the intended learning in the Australian Curriculum.

Assessment must focus on assessing the intended learning rather than just what is currently being taught – we are assessing each student’s progress towards the desired goals. To assess the deep understandings of mathematics, as described above, teachers must ensure that they have taught the key ideas to all students using a high expectations approach and differentiation strategies as described in Good Teaching: Differentiated Classroom Practice. This may include the use of extending or enabling prompts for tasks/problems to broaden access to the learning.

A good source of assessment questions for students in Years 3, 4 and 5 is some of the questions in past NAPLAN Numeracy test papers for Years 3 and 5. It is also important to expose Year 6 students to the types of questions they will meet in Year 7 test. The questions generally align closely with the definition of numeracy and:

- provide no hints about the mathematics children should choose to use
- are for the most part written in contexts so that children have to understand these before choosing the mathematics to use.

Teachers might for example, select questions related to estimating and calculating with whole numbers from the past NAPLAN tests available through the NAPLAN Toolkit and uses them as a pre-assessment before planning explicit teaching in this key element. Similar questions may then be used as a post-test after the teaching sequence.

Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: Will some maths help here? What maths are we using here?

In Science lessons when students are learning inquiry skills, they might use contexts that generate data over time e.g. growth of plants. Numbers are needed for counting, measuring, comparing and ordering. You would point out to students that: We are using some maths here.

In History students are learning to apply their knowledge of numbers to order from first to last (using number lines/timelines), distinguishing between past, present and future (sorting and ordering times and dates) and posing questions about the past from a range of sources and exploring these sources (collecting data).

Questions for reflection

- What is my understanding of the benefits for estimation and mental calculation of being able to partition numbers?
- How do I model the importance of using my brain as the first choice in calculation?
- How do I explicitly teach students mental strategies? Do I model their use by talking my thinking out aloud and explicitly teaching strategies?
- Do I provide time to focus on mental strategies in my class as part of my planning? Is it enough time when this strategy is the one my students will need the most in life?
- What is my understanding of why teaching standard algorithms might be harmful for my students? What does the research tell me?
- Why do I teach standard algorithms? Is this reason valid, given the research evidence? How do I model and teach the skills of estimating with money?
- In what ways do we collect data to monitor progress for each student?
B. Recognising and using patterns and relationships

Key messages

Some may wonder why ‘Recognising and using patterns and relationships’ is important in Numeracy or for that matter mathematics. Mathematics is the science of patterns (viii) and patterns are used to help us organise and make sense of the world in which we live. We therefore need to be able to generalise about these patterns rather than having to study each separate pattern. ‘Mathematics brings to the study of patterns an efficient and powerful notation for representing generality and variability, and for reducing complexity – algebra.’ (ix)

There are patterns in our number system (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 ….) for example, that help students learn to count. There are patterns in our lives (Monday, Tuesday, Wednesday ….) that help us organise our lives and our work. We can learn the words of songs more easily for example, by considering the patterns in the words. In mathematics, learning to count is made easier by recognising the patterns that exist in our number system.

Patterns can be related to each other in ways that can help us study the patterns regardless of the specific elements in the pattern. This is why being able to generalise is so important; it is about asking: ‘What is happening here that has nothing to do with the specific numbers used?’ In a pattern like 4, 8, 12, 16 …., we can say: It is growing by four each time and this generalisation would be the same if the number pattern were 3, 7, 11, 15 … or 29, 33, 37, 41 ….

For example, in Section A of the K–2 booklet, we learned that 3 x 4 = 4 x 3 and that this relationship always works no matter what numbers you use. This can therefore be generalised algebraically as: $a \times b = b \times a$ where ‘$a’ and ‘$b’ can represent any two numbers.

Teachers build on students’ early learning about patterns to ensure that they:

- Understand number patterns and can generalise about them.
- Can recognise and describe the variation (or ‘change’) in quantities and can represent this change in various ways.
- Can write rules to represent variation and relationships in order to identify trends.

Number patterns can be repeating patterns (e.g. 3, 2, 4, 1, 3, 2, 4, 1, 3, 2 ….) which are particularly useful in teaching students to generalise. Or they can be growing patterns (e.g. 2, 4, 6 ….) which describe variation and underpin relationships and functions. If we represent patterns using numbers we can make it easier to both see a pattern and to work with it.

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(viii) Steen, 1988
(ix) Perse, 2003
Recognising and using patterns and relationships

In the *Australian Curriculum: Mathematics* the content descriptors in the Number and Algebra strand describe what students are expected to be able to do by the end of Years 3–6. The numeracy learning continuum has a similar focus on patterns.

### Relevant *Australian Curriculum: Mathematics* Content Descriptors

<table>
<thead>
<tr>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Patterns &amp; algebra</strong></td>
<td><strong>Patterns &amp; algebra</strong></td>
<td><strong>Patterns &amp; algebra</strong></td>
<td><strong>Patterns &amp; algebra</strong></td>
</tr>
<tr>
<td>Describe, continue, and create number patterns resulting from performing addition and subtraction</td>
<td>Explore and describe number patterns resulting from performing multiplication</td>
<td>Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction</td>
<td>Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence</td>
</tr>
<tr>
<td>Solve word problems by using number sentences involving multiplication or division where there is no remainder</td>
<td>Solve word problems by using number sentences involving multiplication or division where there is no remainder</td>
<td>Use equivalent number sentences involving multiplication and division to find unknown quantities</td>
<td>Explore the use of brackets and order of operations to write number sentences</td>
</tr>
<tr>
<td>Use equivalent number sentences involving addition and subtraction to find unknown quantities</td>
<td>Use equivalent number sentences involving addition and subtraction to find unknown quantities</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Extracts from the *Australian Curriculum: Mathematics* Achievement Standards

| Students continue number patterns involving addition and subtraction | Students describe number patterns resulting from multiplication. They continue number sequences involving multiples of single digit numbers | Students continue patterns by adding and subtracting fractions and decimals | Students describe rules used in sequences involving whole numbers, fractions and decimals. They write correct number sentences using brackets and order of operations |

### Australian Curriculum Numeracy Learning Continuum

<table>
<thead>
<tr>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recognise and use patterns and relationships</strong></td>
<td><strong>Recognise and use patterns and relationships</strong></td>
</tr>
<tr>
<td>Identify and describe trends in everyday patterns</td>
<td>Identify and describe pattern rules and relationships that help to identify trends</td>
</tr>
</tbody>
</table>
Recognising and using patterns and relationships

The verbs ‘identify’ and ‘describe’ as used in the numeracy learning continuum are consecutive; you can’t describe without having first identified a pattern. Some students might need to be supported to ‘identify’ a pattern and some may need to be taught the words to communicate what they see.

Teachers can help students to represent patterns using numbers through activities such as when students examine patterns in actions: click, click, stamp, stamp, jump, click, click, stamp, stamp …. or in colours: red, red, blue, blue, green, red, red …. They can see the pattern and be encouraged to say: ‘There are two of one thing, then two of another thing, then one of another thing, then two of the first thing ….’ Using this language they can see that the two patterns above are the same, even though they are represented in different forms. What is the same is the number of times the clicks or colours occur and students can then represent both patterns with other objects and materials.

Planning

Teachers use the information in the ‘Links to the curriculum’ table to plan by backward mapping from the expected outcomes. To plan effectively, teachers also need to have some understanding of what their students already know, understand and can do through pre-assessments and other ongoing formative assessments. Teachers should refer to Good Teaching: Quality Assessment Practices so that they can build on this prior learning with the intended 3–6 learning about patterns and relationships.

The following activities describe the sorts of learning opportunities teachers need to create to develop the required, stated learning both for numeracy and the mathematics that underpins it.

In the ‘Recognising and using patterns and relationships’ element of the numeracy learning continuum, the main understandings that Year 3–6 students need to learn are:

- An understanding of number patterns and being able to generalise about them (including trends).
- An ability to identify and write rules that represent relationships in the growth of patterns in order to identify trends.

Putting it into practice

1. Teaching students to understand patterns and to generalise about them using numbers

There are some words that students need to learn in order to talk about and describe patterns. These need to be modelled by their teacher and explicitly taught. The terms include:

- **Element** of a pattern
- **Position** of an element in a pattern
- **Term** in a pattern
- **Cycle** of a pattern
- **Rule** relating two patterns

The ‘elements’ are the fundamental building blocks of the pattern. In the pattern C, B, D, C, B, D …. the elements are C, B, and D while the cycle is C, B, D since these three elements repeat in that same order. In the pattern X, X, O, X, O, X, O, X, O, X …. the elements are X and O while the cycle is X, X, O, X, O because after this cycle of elements the pattern repeats. The ‘position’ of an element is numerical and describes the position of the element in the pattern sequence; it can be the first element, the second element (or the first repeat of the first element), the third element (or the second repeat of the first element), and so on.

In a ‘growing’ pattern each new element of the pattern is called a **term**. For example, in the pattern 4, 7, 10, 4, 7, 10 …. 7 is the second term and 10 is the third term. This language is useful when you want to ‘predict’ the fourth element of the growing pattern for example, based on the rule of: ‘it is growing by three each time’. Generally, a growing pattern needs three terms to indicate the nature of the pattern.
Growing number patterns can be created using operations such as addition, subtraction and multiplication. For example, by starting at a number and adding, subtracting or multiplying the same amount to each successive term, the following patterns can be created:

1. 3, 6, 9, ... this pattern is growing by adding 3 to each successive term
2. 62, 73, 84, ... this pattern is growing by adding 11 to each successive term
3. 80, 76, 72, ... this pattern is decreasing each successive term by 4 (or growing by -4)
4. 2, 4, 8, 16, ... this pattern is growing by multiplying each successive term by 2
5. 8, 4, 2, 1, ½, ... this pattern is growing by multiplying each successive term by ½

Students need to identify the operation being used and the number being added, subtracted or multiplied. When they can do this they are in a position to predict the next term in a pattern. For pattern 3 above for example, they can answer the question: What will the sixth term of the pattern be? They determine this using a mental calculation or writing the pattern down:

<table>
<thead>
<tr>
<th>Term</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80</td>
<td>76</td>
<td>72</td>
<td>68</td>
<td>64</td>
<td></td>
</tr>
</tbody>
</table>

They look at the numbers in the pattern and determine that the ‘difference pattern’ is ‘take four from the last term’ and hence know that the next term will be 68 and the one after will be 64.

By predicting successive terms students are in a position to identify trends in patterns. A trend is a tending towards something. A pattern can:

- continue to increase; trend upwards
- continue to decrease; trend downwards
- continue to stay about the same; steady trend

For example, consider the following number patterns, their nature and trending behaviour:

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Nature</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 6, 9, ...</td>
<td>Increasing</td>
<td>Upwards</td>
</tr>
<tr>
<td>10, 5, 2.5, 1.25, ...</td>
<td>Decreasing</td>
<td>Downwards</td>
</tr>
<tr>
<td>2.5, 4.5, 6.5, ...</td>
<td>Increasing</td>
<td>Upwards</td>
</tr>
<tr>
<td>3, 4, 3, 2, 3, 4, ...</td>
<td>Up and down</td>
<td>Steady/Constant</td>
</tr>
</tbody>
</table>

Students should be able to look at the numbers in a pattern, or in the numbers used to describe a pattern, and decide whether it is trending upwards, downwards, or is steady.

Some activities for learning about patterns and relationships

1. Have students work in pairs to write number patterns. One thinks of a starting number, the other thinks of the operation and difference pattern (e.g. add five, take seven, and multiply by two). The first writes the starting number and the second writes the next two terms. The first has to guess the difference pattern; if they get it right they swap over and do the activity again, this time the winner thinks of the starting number.

2. In pairs, get students to draw a caterpillar. The caterpillar is made of circles with designs on them in a repeating pattern, for example:

Each pair hold up their pattern and ask the children what the next circle in the pattern will look like. Other children in the class draw it. The teacher asks the children with the correct ‘element’: What is the repeating part or cycle of the pattern? Children have to draw the repeating part, or cycle.
3. Give students a range of patterns in number and describe each of them as 'growing' or 'repeating'. They then have to explain to those in their group why they think they are growing or repeating, and if repeating, what the pattern cycle is. Possible patterns might be:

Fee, fi, fo, fum, fee, fi, fo, fum ….

1, 1, 2, 3, 5, 8 ….

4. Provide cubes for students to make a growing pattern that represents a caterpillar that is growing every day:

2 days old 4 days old 6 days old

Ask students what is happening to their caterpillar. They should say things like: 'Each day the caterpillar grows a half a cube' or: 'very two days the caterpillar is another cube longer'.

Ask students questions such as: How long will the caterpillar be after five days? When the caterpillar is ten cubes long, how old will it be?

5. Give pairs of students a packet of headless matches. Have children build the following pattern using their matches:

Ask the following questions:
- How is your pattern growing?
- Can you suggest a difference pattern describing the growth? e.g. add three matches each time.
- How many matches would it take to make 5, 10, 15 sections of the pattern? Build these and see if you were right.

6. Have students work together to solve word problems by using number sentences e.g. Bella needs to buy 18 party hats; they come in packets of four. How many packets will she need?

Students will develop a table which sets out the number of party hats in each packet:

<table>
<thead>
<tr>
<th>Number of party hats</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of packets</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

They are then able to see the growing pattern as ‘add four each time’ and can see the solution to the word problem at a glance by setting the problem out this way. They can then decide that Bella needs to buy five packets, even though she will have two party hats left over, because she can’t buy half a packet.

A similar word problem using division might be:

There were some cupcakes on a plate. Fiona took half of them, then Sam took half of those remaining and Matt took half of what was left. There were now two cupcakes on the plate. How many were there to start with?

<table>
<thead>
<tr>
<th>Cupcakes on the plate</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>People taking cupcakes</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(Fiona)</td>
<td></td>
<td>(Sam)</td>
<td>(Matt)</td>
<td></td>
</tr>
</tbody>
</table>

By setting the problem out in a table students can ‘see’ the pattern is ‘half off each time’ or ‘divide by two’. They are then able to work out that there must have been 16 cupcakes on the plate.
2. Ability to identify and write rules that represent relationships in the growth of patterns in order to identify trends

If students are to understand trends they need to understand the relationships between elements of the pattern and the number of the element in the pattern. Whereas a ‘difference pattern’ might be a simple statement of what they can see is happening e.g. ‘add two each time’ or ‘multiply by ½ each time’ this does not describe the relationship between the element in the pattern and the position of the element in the pattern.

For example, in the pattern below, made by adding three matches each time

we can draw a table showing the values in the pattern as:

<table>
<thead>
<tr>
<th>Position number (element in the pattern)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
</tr>
</tbody>
</table>

Although the difference pattern is ‘add three’ it does not describe the relationship between the position of the element in the pattern and the number of matches in the element. This makes it very difficult to predict the number of matches in say, the 8th or 20th element of the pattern, without having to make or draw these patterns and counting the number of matches.

If we can identify the relationship between the position number and the number of matches in the pattern, this makes it a lot easier to predict the number of matches in any element in the pattern.

To do this we need to look very carefully at the two sets of numbers, and in particular the 1 and 4, 2 and 7, 3 and 10 and so on: What is the relationship between the numbers in the top row and the numbers in the second row each time? Is there something happening that has nothing to do with the actual numbers?

What is happening to 1 to get 4? What is happening to 2 to get 7? Is it the same each time?

The simple difference pattern that we used to determine the numbers in the second row ‘add three’, will usually provide a hint if the pattern is growing at a constant rate: there will be a ‘three’ in the rule.

We can see that if we ‘times the position number by three and then add one’ we will get the number of matches used. So:

\[
\begin{align*}
1 \times 3 + 1 &= 4 \\
2 \times 3 + 1 &= 7 \\
3 \times 3 + 1 &= 10 \\
4 \times 3 + 1 &= 13
\end{align*}
\]

The relationship in words is: ‘Multiply the position number by three and add one’.

We are now able to predict the number of matches for any element in the pattern without having to build the pattern and count them.

For example, we can find the 10th term in the pattern by multiplying the position number (10) by three and then adding one; the 10th term is \(10 \times 3 + 1\) which is 31.

We can find the 20th term in the pattern by multiplying the position number (20) by three and then adding one; the 20th term is \(20 \times 3 + 1\) which is 61.
Some further activities for developing understandings about patterns

- Students use the patterns following (1. and 2. below) and construct a table to show the pattern numbers. For example:

<table>
<thead>
<tr>
<th>Position number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of squares</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- For each table they describe what is happening and propose a difference pattern which could be used to find the next numbers. For example: ‘For each new element in the pattern there are an extra three squares so the difference pattern is ‘add three squares’.

- For each table they predict the next numbers in the table, without drawing or building, using their difference pattern.

- Students look closely at the numbers in the top row of their table and the numbers in the bottom row of their table and write their rule in full so it can be used for prediction e.g. number of squares = position number x 3.

- Students use their rule to predict the number of squares for any position number (without building or drawing the pattern).

- Students write a statement about how the pattern in each case is trending; is it increasing, decreasing or stable?

Patterns

1.

2.

3. Students could go to the local cemetery and obtain dates of the births and deaths from each tombstone. They tabulate this data in a table such as:

<table>
<thead>
<tr>
<th>Birth year</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Death year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age at death</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

They consider the data in small groups or pairs and identify any patterns or trends in their data, summarising using statements such as:

- It seems that people born before 1900 did not live longer than forty years.

- It seems that the more recently people were born the longer they lived, and so on.

4. Students plant some seeds in Science and watch them grow, measuring the height every day and recording the heights in a table such as those above. They can decide whether the numbers indicate their plant is growing or repeating, trending upward or steady or decreasing and predict what the next day’s measurement will be.
Monitoring and assessment

Most of the ideas presented in these activities can be used to assess the understanding and monitoring of pattern and relationships. Note that understanding of these concepts needs to include higher-order reasoning about pattern and relationship; teaching and assessment must go well beyond asking students to continue and repeat given patterns. Students should be able to make conjectures about their patterns and predict what will happen to a pattern as it trends upwards or downward, based on the numbers in their pattern. For example: What will happen to a pattern in which each element is half the next? Will it ever reach or pass zero?

To monitor this learning, teachers keep a record of whether students can first identify a pattern; recognise its difference pattern and the operation responsible; hypothesise a rule in words and then use the rule to predict terms beyond the pattern; and describe the trend of the pattern – in that order – as they progress, focusing support on the actions they are unable to master.

Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: Will some maths help here? What maths are we using here?

In Science lessons when students are learning inquiry skills they might see the usefulness of generalising patterns (e.g. you plant seeds, they germinate, they grow a bit each day, they develop fruit and so on), so that they don’t have to describe what happens for each single plant.

In History students are learning to apply their knowledge of time by sequencing objects and events (using number lines/timelines) and identifying patterns in history from the past, present and future (sorting and ordering time and dates). They can connect sequenced dates with events and describe any trends they might see over time: Are populations in certain countries growing? What makes us think that? Are they stable or increasing? How do we know?

In English students are learning about how word families are related for spelling and writing. They learn the patterns used to organise texts such as page numbering, tables of content, headings, chapters, and so on. Titles and headings can be used to predict the content and genre of texts and students learn that information texts have different patterns of layout and navigation from fiction.

Questions for reflection

• Do I know what number patterns are and why they are included in the Australian Curriculum: Mathematics and numeracy learning continuum? Could I explain this to another teacher?
• How well can my students describe a pattern?
• Can students show me one pattern in another form? What is their understanding of the difference between a growing pattern and a repeating pattern?
• How well do students know and use the language they need to describe and compare patterns? What vocabulary might I need to focus on and explicitly teach?
• How well do students predict what will happen when they continuously add or subtract the same number? What about if they multiply each successive term by the same amount?
• How do I recognise when students have learned to generalise about a pattern? What should I listen for?
• What is a trend and how might I describe it to colleagues and students?
• In what ways do we collect data to monitor progress for each student?
C. Using fractions, decimals, percentages, ratios and rates

Key messages
It is important to consider the order of the words in this heading. Decimals, percentages, ratios and rates are all different forms of fractions. They are also developmentally cumulative, so that students learn about common fractions first, then about decimal fractions, then about percentages – which are a particular type of fraction, having a denominator of 100; then about ratios and rates.

The second thing worth noting in the heading is the word ‘using’. Note that you can’t use fractions if you don’t first understand them. So in order for students to choose to use fractions in situations that demand their use, they need to have a deep understanding of what fractions are and what they can be used for. Many Australian students – even in secondary settings – don’t really understand fractions. Research(x) has revealed students in Year 8 who thought that a fraction was merely a different way of writing two whole numbers, separated by a ‘funny little line’.

It is critical to explicitly teach the idea of ‘equal quantities or parts’ to students in the early years. This is because most young children come to school with a ‘common’ understanding of fractions that is underpinned by the notion of parts or pieces of a whole but they don’t necessarily understand that the parts have to be exactly the same amount or quantity. For example, Mum might say ‘half each’ to two children or ‘have a quarter each’ to four brothers and sisters. This does not mean ‘measure it exactly and make sure you get the same amount each’; it usually just means ‘share it as evenly as you can’.

It is appropriate in the context of the home or community to use this ‘common’ terminology. However, since children bring this understanding to the study of fractions at school, teachers need to explicitly build onto the knowledge of halves being two parts and quarters being four parts, the idea of equal parts.

These understandings are the basic ‘building blocks’ of fractional thinking. Teachers of Year 3 in particular are about to start teaching students how to represent fractions using symbolic notation (a numerator and a denominator). It is critical that students understand fractional amounts in words before they are learned in symbols.

Links to the curriculum
The Australian Curriculum: Mathematics recognises that fractions can be either common fractions, or decimal fractions. Common fractions are fractions represented symbolically as a numerator (describing quantity) divided by a denominator (describing the number of equal parts the whole is broken into). The ‘funny little line’ is actually called the vinculum and is a different representation for division. Decimal fractions are those fractions with a denominator of 10 or multiples of 10. These fractions e.g. 1/10, 3/100, are usually written as the fractional part of a decimal number e.g. 1/10 = 0.1, 3/100 = 0.03. Because they are fractional parts of a whole number they are part of the ‘Fractions and Decimals’ row of the Australian Curriculum: Mathematics. For greater clarity the descriptors in the ‘Fractions and Decimals’ row have been separated into common fractions and decimal fractions.

x Perso, T., 1989
### Relevant Australian Curriculum: Mathematics Content Descriptors

<table>
<thead>
<tr>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common fractions</strong>&lt;br&gt;Model and represent fractions including (\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}) and their multiples to a complete whole</td>
<td>Investigate equivalent fractions used in contexts&lt;br&gt;Count by quarters, halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line</td>
<td>Compare and order common unit fractions and locate and represent them on a number line&lt;br&gt;Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator</td>
<td>Compare fractions with related denominators and locate and represent them on a number line&lt;br&gt;Solve problems involving addition and subtraction of fractions with the same or related denominators&lt;br&gt;Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies</td>
</tr>
<tr>
<td><strong>Decimal fractions</strong>&lt;br&gt;Recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation</td>
<td>Recognise that the number system can be extended beyond hundredths&lt;br&gt;Compare, order and represent decimals</td>
<td>Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers&lt;br&gt;Multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without digital technologies</td>
<td></td>
</tr>
</tbody>
</table>

### Extracts from the Australian Curriculum: Mathematics Achievement Standards

| Students model and represent unit fractions | Students recognise common equivalent fractions in familiar contexts and make connections between fraction and decimal notations up to two decimal places | Students order decimal and unit fractions and locate them on number lines<br>They add and subtract fractions with the same denominator | Students connect fractions, decimals and percentages as different representations of the same number<br>They solve problems involving addition and subtraction of related fractions. Students locate fractions and integers on a number line<br>They calculate a simple fraction of a quantity. They add, subtract and divide decimals and divide decimals where the result is rational<br>They calculate common percentage discounts on sale items |
**Planning**

Teachers use the information in the ‘Links to the curriculum’ table to plan by backward mapping from the expected outcomes. Because the expectations are described in terms of what students should be able to know, do and understand, teachers need to ask: What do I need to do to enable them to do and know these things?

To plan effectively, teachers also need to have some understanding of what their students already know, understand and can do through pre-assessments and other ongoing formative assessments. As fractions, decimals and percentages areas where many students hold misconceptions it is important to know where they are in their learning through diagnostic tasks and to target teaching to overcome these possible misconceptions.

The activities in the following ‘Putting it into practice’ section describe the sorts of learning opportunities for fractions, decimals, percentages, ratios and rates teachers need to create to develop the required, stated learning, both for numeracy and the mathematics that underpins it. Your students might need more or less reinforcement, depending on where they ‘are at’ in their current learning.

Key ideas about fractions in levels 3 and 4 are:

- Learning to represent fractional amounts in numerator-denominator format.
- The idea of ‘counting fractions’ in order to deepen understandings of the different roles of the numerator and denominator.
- The understanding that the same fractional amount can have different names.

- Fractions, including decimal fractions, can be compared and ordered, represented on a number line and used for calculation.
- We can find a fraction of a quantity.

The headings in the numeracy learning continuum for this element are ‘Interpreting and applying proportional reasoning’, even though the descriptors that follow for Years 3–6 do not seem to deal with proportion. Since proportional reasoning is about the relationship between the whole and its parts, it is important that teachers know about this concept early in the curriculum — when fractions are being introduced — rather than later, so that students can build the deep understandings needed to be good at proportional reasoning later.

**Proportional reasoning**

A proportion is a part or quantity of a whole considered in comparison to the whole. **Proportional reasoning** refers to the thought processes required to make statements about proportions, such as ‘half as much as’ or ‘double the amount’ or ‘three times greater’. These statements provide more comparative detail than simply ‘more than’ or ‘less than’. So it is more than understanding comparison; it is about understanding the **context or situation** that in which the comparison is occurring.

Proportional reasoning is more than being able to calculate ratio. It is very complex and depends on many interconnected mathematical ideas which are developed over many years. These start with multiplicative thinking, which is when students think of 10 as ‘two fives’ (or five twos), rather than ‘one more than 9’ (additive thinking). It is important that we ‘build in’ these complex ideas in mathematics learning from an early age rather than beginning to pay attention to them when students are learning about rates in the secondary years.
An example of proportional reasoning is required when considering the following question: Is a half of a 12-square block of chocolate the same as one third of an 18-square block of chocolate?

In an absolute sense, the answer is: If the chocolate squares are the same size, then they are the same. In a relative sense the answer is that: The half of the 12-block is more than a third of the 18-block; from the perspective of the whole blocks, half is bigger than one third. So, proportional reasoning is often about comparing a proportion to its whole amount, as well as comparing it to a proportion of another whole. This requires mental gymnastics! It is complex and therefore requires deep understandings about fractions and the wholes they relate to. In calculating, we mostly try to reduce the wholes to their smallest common units.

Proportional reasoning is also needed to determine the ‘best buy’ when 1 kg costs $3.50 and 1.5 kg costs $5.20, and working out whether ‘4c per litre off’ is better than ‘5% off the total’ when purchasing petrol.

More will be said about these ideas in the 7–10 booklet. Suffice it to say that Year 3–6 teachers need to be aware of the significance of early fractional understandings in the development of proportional reasoning for future mathematics study and for numeracy.

Putting it into practice

**Common fractions**

In the early years, students learned that one half is ‘one out of every two equal parts’. They learned to say this to reinforce their understanding of what one half is. When teachers introduce symbolic representation of the words ‘one half’ they need to continue to stress the importance of students reading $\frac{1}{2}$ as ‘one out of two equal parts’. They should also be able to do this for one quarter:

<table>
<thead>
<tr>
<th>In objects or drawings</th>
<th>In words</th>
<th>In symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One out of two equal parts, or one half</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>One out of four equal parts, or one quarter</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

**The symbolic form**

In learning the symbolic representation of fractions it is important for teachers to know that students must learn about denominator first. The denominator describes the number of equal parts the whole is partitioned into. When students say ‘one out of four equal parts’ or, if there is more than one whole, ‘one out of every four equal parts’ (see the K–2 booklet) this helps them understand what the symbolic form $\frac{1}{4}$ represents.

You will notice that in the Year 3 entry of the *Australian Curriculum: Mathematics*, the content descriptor – ‘model and represent fractions including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$ and their multiples to a complete whole’ – focuses entirely on denominator (as indicated by including unit fractions only). Students have a whole year to learn and understand this concept; numerator is an outcome for Year 4.
Activities for consolidating this concept might include drawing 2, 3, 4, 5, or 10 of the same shape and shading one of them; or shading 1 of 2, 3, 4, 5, or 10 different shapes and shading one of them. This helps students to learn that fractions can be of a collection (which is the whole), as well as a single shape or object (which is the whole).

Teaching can then quite naturally move into shading two or three of the shapes or sections by asking students: How many thirds are shaded now? How many fifths are shaded now? How many halves are shaded now? As students develop these understandings, they can be challenged by questions such as: How many more fifths do we need to shade, to shade the whole shape? How many more quarters do we need to shade, to shade the whole shape? Jane, come and shade another third — how many thirds are shaded now? Sam, come and shade another two tenths — how many tenths are shaded now? It is important that students are given many opportunities to partition shapes themselves rather than shading pre-partitioned shapes. Initial experiences with partitioning will also involve folding and cutting shapes e.g. folding A4 paper to show halves, quarters, thirds, fifths and tenths and seeing the relationships between them.

While fractional understandings are being developed through these activities, it makes sense to ask students questions such as: What if you have three of the thirds shaded — is there another name for that? Is three thirds the same as one whole? Is five fifths the same as one whole? If you shade six fifths is that one and one fifth? How might you write that? What about ten fifths? Is that two wholes?

These questions and activities will build understandings of unit fractions (those with one as numerator) and denominator: Students will soon learn for themselves that changing the number on top (the numerator) is about counting; the numerator tells you how many of the equal parts you have. And, that if the number of parts (numerator) is the same as the number of equal parts that the whole is made up of, (the denominator) then it represents a whole. If it represents a whole and a fraction e.g. 1½, then it is called a mixed number or mixed numeral since it is made up of a whole number and an additional fractional part of another whole of the same amount.

This understanding paves the way for an entirely new learning activity: that of counting fractions.

In the same way that students learn to count whole numbers, they can also count fractions of a whole number, and say, for example: one third, two thirds, three thirds, four thirds, five thirds, etc. This counting can be supported initially using a number line, so that students are skip-counting in thirds and hence can visualise what they are doing. By marking the number line with ones (or wholes) they can see that there are three thirds in a whole:

```
0 —— 1 —— 1 1/3
```

You can prompt students as they are counting to give the different names for the whole numbers and mixed numbers e.g. one third, two thirds, three thirds (or one), four thirds (or one and one third), five thirds (or one and two thirds), and so on.

Students enjoy counting in fractions, one student at a time, forwards or backwards, each saying the next fraction and its many names.

By skip-counting in fractions along a number line and using the same markings for the whole numbers (0, 1, 2, …) students can see and learn that fractional numbers can be ordered; they can see, through various activities, that 1/5 is less than 1/4 and that 1/4 is less than 1/3. This visualisation and drawing of fractions with different denominators helps students to understand the relative sizes of fractional amounts, that — contrary to what they might expect based on their understanding of whole numbers — the larger the denominator the smaller the fractional quantity. By visualising one third on a number line, they can intuitively determine that ½ is between ⅔ and 1, and ⅔ is just a little bit less than 1 since that is 7⁄6, and so on. They are able to compare fractions using a number line and can order fractions and justify their position based on what they know about fractions; comparing 3/5 with 2/3 becomes an easy task using a number line.

This benchmarking of fractions between those they can ‘see’ and including whole numbers, will help students to estimate the relative sizes of fractions. They can picture ⅔ as being between ⅔ and 1 and ⅔ as being slightly larger than ⅔.
Similarly, knowing that \( \frac{3}{5} \) and \( \frac{1}{4} \) are all different names for \( \frac{1}{3} \) they are learning the fundamental understanding of equivalent fractions; that fractions can represent the same amounts even if they have different names. They can picture \( \frac{1}{4} \) on a number line and know that it is half of \( \frac{1}{2} \) which is also \( \frac{1}{3} \). This leads to the understanding that \( \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \) and \( \frac{3}{4} + \frac{1}{4} \) is 1. By deeply understanding fractions in this way they are able to estimate \textbf{fractional quantities} before applying written methods when calculating with them.

\[ \begin{align*}
  \text{Some activities for learning about fractions} \\
  &\text{• Give students some number lines and have them work in pairs to draw worms on the number line: three and one half units long; six and two tenth units long; five and seven eighth units long. Have them determine which of these worms is longest and why?} \\
  &\text{• Have students count apples in fractional amounts i.e. first cut them into quarters, thirds, halves. Then ask children to work together to determine how many apples were cut up to give that many halves, thirds and quarters.} \\
  &\text{• Have students work in groups to ponder problems such as:} \\
  &\quad - \text{I bought a muesli bar and asked you whether you wanted one half, one third or one quarter. Which of these amounts would give you the most chocolate? Show why.} \\
  &\quad - \text{Mary spent one half of her pocket money and James spent one half of his pocket money? Did they spend the same amount? Give examples.} \\
  &\quad - \text{Which is the bigger fraction: } \frac{1}{3} \text{ or } \frac{2}{4} \text{? How do you know? Draw a diagram to show you are right.} \\
  &\quad - \text{Which of these is closest to 2 whole numbers in value? } \frac{1}{3}, \frac{3}{4}, 1 + \frac{1}{4}, \frac{10}{4} \\
  &\text{• Make collections of environmental materials such as leaves, rocks etc. When students come back to class ask them to write fraction questions in words about their collections for other students to solve e.g. There are six leaves in my collection. If I give half of them to Sienna how many will I have left?} \\
\end{align*} \]

\[ \begin{align*}
  &\text{• Give students working in pairs a number line marked only with whole numbers but with arrows pointing at other places on the number line, marked with letters: } A, B, C \ldots \text{. Have them work together to estimate the fractions represented by the letters. They then pass the number line onto the next pair, and compare answers and work them out together.} \\
  &\text{• Have students work from the fraction to the whole:} \\
  &\quad - \text{is one third of a shape. What might the ‘whole’ look like?} \\
  &\quad - \text{is } \frac{3}{4}; \text{ what might the ‘whole’ look like? Are there any other ways it could look like? Draw them.} \\
\end{align*} \]

These understandings enable students to find fractional amounts of whole numbers. If they have learned that \( \frac{1}{3} \) is ‘one out of three equal amounts’ then, having learned about numerators, they are now able to say that \( \frac{3}{4} \) is ‘two out of three equal amounts’; and that to share two wholes among three people, is each of the wholes shared equally among the three people.

Share two oranges among three people is one orange shared among three people equally i.e. \( \frac{1}{3} \) each and the other orange shared among three people equally, \( \frac{1}{3} \) each; so each person has two thirds.

Two wholes shared equally among three people: \\
\[ \text{is one third of each whole, for each} \]

One whole shared among three people \( = \) one third each, \\
Two wholes shared among three people \( = \) one third of one whole + one third of the other whole = \( \frac{2}{3} \)

Symbolically, \( 2 + 3 = \frac{2}{3} \)

This helps students to understand that the vinculum (the little line between the numerator and denominator) is a different way of writing a division symbol.
• Students need to think of a fraction as a division: \(\frac{3}{4}\) is \(2 \div 3\), \(\frac{1}{5}\) is \(1 \div 5\), \(\frac{3}{5}\) is \(3 \div 4\). They can learn this by completing tables such as:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>(\frac{1}{2})</th>
<th>(\frac{1}{4})</th>
<th>(\frac{3}{5})</th>
<th>(\frac{1}{5})</th>
<th>(\frac{35}{100})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Division</td>
<td>(1 \div 2)</td>
<td>(3 \div 4)</td>
<td>(2 \div 3)</td>
<td>(3 \div 5)</td>
<td>(7 \div 10)</td>
</tr>
</tbody>
</table>

Students work in pairs to share: two oranges equally between three people, four apples shared equally between five people, six pizzas equally between four people, three oranges equally between four people/two people/five people, and so on. They should be encouraged to write these problems in different ways such as:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Symbolically</th>
<th>As a fraction</th>
<th>For one</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share three oranges equally between four people</td>
<td>(3 \div 4)</td>
<td>(\frac{3}{4})</td>
<td>Share one orange between four people = (\frac{1}{4})</td>
</tr>
<tr>
<td>Share one apple between five people = (\frac{1}{5})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For three oranges: (3 \times \frac{1}{4} = \frac{3}{4}) of an orange each</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem</th>
<th>Symbolically</th>
<th>As a fraction</th>
<th>For one</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share four apples equally between five people</td>
<td>(4 \div 5)</td>
<td>(\frac{4}{5})</td>
<td>Share one apple between five people = (\frac{1}{5})</td>
</tr>
<tr>
<td>For four apples: (4 \times \frac{1}{5} = \frac{4}{5}) of an apple each</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Decimal fractions**

When students have these deep fractional understandings and know that they can skip count from 0 to 1 in tenths by skipping 10 equal places, they can be taught that \(\frac{1}{10}\) can also be represented by 0.1 and that 2.1 for example, is the same as two wholes and 1 tenth. They are learning that 0.1 = \(\frac{1}{10}\) and that tenths are special fractions because they link into what they already know about ones, tens and hundreds. They can cut a single straw into ten equal parts and lay them end to end to show that ten tenths is the same as one whole (and can also show this on their number line). You can ask them to use straws to show you 1.2 or 3.5 or 0.2 and to also represent these using mixed numbers:

| Decimal number | 0.2 | 0.6 | 1.1 | 2.3 | 4.5 | 5.1 | 7.6 | 10.3 |
| Mixed number   | -   | -   | 1\(\frac{1}{10}\) | 2\(\frac{3}{10}\) | 4\(\frac{3}{10}\) | 5\(\frac{1}{10}\) | 7\(\frac{6}{10}\) | 10\(\frac{3}{10}\) |
| Fraction       | \(\frac{2}{10}\) | \(\frac{6}{10}\) | \(\frac{11}{10}\) | \(\frac{23}{10}\) | \(\frac{45}{10}\) | \(\frac{51}{10}\) | \(\frac{76}{10}\) | \(\frac{103}{10}\) |

When students understand this relationship they are ready to learn about **hundredths**. Place ten tenths of a straw in front of you and ask: ‘What will happen if cut each of these tenths into ten equal pieces?’ Attempt to do this using a sharp knife (note that they don’t have to be exactly one tenth of the size). Students should understand and be able to say: **There will be one hundred tiny pieces of straw.** Explain to them that each of these is **one hundredth of one whole** i.e. one hundred of these put together will make one whole straw.
Show students how we write one hundredth as a fraction as $\frac{1}{100}$ and as a decimal, 0.01. You can ask them: *How many hundredths are there in one whole? One half? One quarter? One tenth? How might we write these as decimal (not fractional) numbers?* Show them:

<table>
<thead>
<tr>
<th>Decimal fraction</th>
<th>0.5</th>
<th>0.25</th>
<th>0.10</th>
<th>0.20</th>
<th>0.75</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common fraction</td>
<td>50/100</td>
<td>25/100</td>
<td>10/100</td>
<td>20/100</td>
<td>75/100</td>
<td>1/100</td>
</tr>
</tbody>
</table>

This understanding provides the perfect bridge to **percentages**; *per cent*, meaning ‘out of one hundred’ means that most decimal fractions can be written as percentages, as can common fractions. Students should be able to make connections between fractions (common and decimal) and percentages as proportions of a whole.

<table>
<thead>
<tr>
<th>Decimal fraction</th>
<th>0.5</th>
<th>0.25</th>
<th>0.10</th>
<th>0.20</th>
<th>0.75</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common fraction</td>
<td>50/100</td>
<td>25/100</td>
<td>10/100</td>
<td>20/100</td>
<td>75/100</td>
<td>1/100</td>
</tr>
<tr>
<td>Percentage</td>
<td>50%</td>
<td>25%</td>
<td>10%</td>
<td>20%</td>
<td>75%</td>
<td>1%</td>
</tr>
<tr>
<td>In words</td>
<td>Fifty per cent, or one half</td>
<td>Twenty five per cent, or one quarter</td>
<td>Ten per cent, or one tenth</td>
<td>Twenty per cent, or one fifth</td>
<td>Seventy five per cent, or three quarters</td>
<td>One per cent, or one hundredth</td>
</tr>
</tbody>
</table>

Comparing decimals can be challenging for some students, especially when they don’t have the understanding of these different representations of the same numbers. As suggested in Section A, it is important that students are introduced to decimals using materials and models other than money as this can potentially lead to misconceptions such as there only being two decimal places in our number system.

### Templates

#### Good Practice

#### Video Tool

#### Conversation Starters

#### Resources

Some further activities for learning about fractions, decimals and percentages

- Have students make cards representing different number equivalents: decimal, fraction and percentage. They can play ‘snap’ or ‘concentration’ or ‘bingo’ to practise recognising the equivalents.

- Have students work in pairs; give each pair a special number e.g. $\frac{1}{2}$, $\frac{3}{4}$, 0.7, 3.4, or 25% and ask them to prepare a chart with as many different representations of that number that they can, including pictures, shapes, collections. These can be hung around the room or on lines with pegs across the room.

- Give pairs of students a strip of paper about one metre long each. Have them fold the strips in halves, quarters, thirds, fifths and any other fractions they can fold. Have them write the names on the strips using different colours for decimals, fractions, percentages and so on. Colour some of the sections of the strips and hang them in the room or on a number line.

- Give students some circles (not already partitioned) and have them shade fractional amounts of the circles e.g. $\frac{1}{3}$, $\frac{3}{4}$, 30%, 90%, 80%. The shaded amount does not have to be exact but they need to be able to justify the amount they have shaded using benchmark amounts e.g. $\frac{3}{4}$, $\frac{1}{2}$.

### Solving problems involving proportions

Many students have great difficulty solving proportional problems. Often this is because they don’t understand exactly what the whole unit is in a problem. Helping them to determine what this is will help to clarify the problem. If the proportion problem involves two or more steps e.g. ‘find the proportional amount and then subtract it from the whole’, or ‘subtract the remaining amount from the whole’, students need to be taught to think...
Using fractions, decimals, percentages, ratios and rates

of this in words i.e. to paraphrase it so they can think about the problem in two steps.

For example:

On a bus there are twenty-four people. One sixth of them are children – how many are adults?

Step 1: What is the whole amount? 24.

Step 2: Find the proportion/fraction/percentage: \( \frac{1}{6} \) of 24 (one out of every six, of 24, is 4) = 4

That is, there are 4 children.

Step 3: How many are adults? 24 people subtract 4 children = 20 adults.

Proportion problems

- Jenny had a piece of ribbon one metre long. She needed to cut one quarter of it for her hair. How long was the hair ribbon?
- Frank gets $2.00 for pocket money. He spent one fifth of his pocket money last week on comics. How much did he have left over?
- There were 25 209 people at a soccer match. One third of the people at the match supported the visiting team. About how many people supported the home team?
- The price of apples is $4.00 per kilogram. One apple weighs about 250 grams. The cost of ten apples will be close to…? About how many apples will weigh a kilogram? About how many apples could I buy for $20.00?
- In a park there are six red-gum trees for every seven blue-gums. There are 56 blue gums. How many red-gum trees are there?
- One in every 40 raffle tickets wins a prize. There are 520 raffle tickets sold. How many tickets win a prize?
- Sarah made 600 grams of fried rice and used one cup of rice. If she uses 1.5 cups of rice, how much fried rice will she make?
- A certain shop advertises ‘30% off everything today’. How much will you pay for a new CD which has a regular price of $20?

Monitoring and assessment

All the tasks and problems outlined can be used for assessment as well as teaching. Students can be asked individually to find solutions and to draw and write about how they know they are correct. You might also give them a question with a hypothetical student’s response, and have them mark it: Are they right? How do you know?

Past NAPLAN questions are useful assessment tools to inform planning and teaching. These questions and associated teaching strategies can be accessed via the NAPLAN Toolkit and all teachers in Years 3–6 are encouraged to use this resource to inform their teaching. The IMPROVE website is an additional tool for formative assessment and also uses past NAPLAN questions.

Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: Will some maths help here? What maths are we using here?

In Science lessons children will need to know how to compare numbers including fractions, and put them in order. They might make up solutions that require proportions such as ‘one teaspoon of sugar for every litre of water’.

In History and Social Sciences students might be comparing composite populations of countries such as ‘one third of the population are Asian’ or ‘two fifths of the country is populated’ and so on. These present opportunities for students to use their mathematical understandings and teachers should prompt them to explain what these statements mean in the contexts in which they are discussed.

In Technology students might be involved in cooking or making craft which requires fractional measurements such as ‘half a cup’, ‘one quarter of a metre’ or ‘two tablespoons of flour to three cups of water’ and so on. Teachers should use these opportunities to challenge the mathematical understandings of their students. They can do this by asking ‘what if’ or ‘let’s suppose’ questions that require ‘twice as much’ or ‘half as much’.

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Good Practice

Video Tool

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Questions for reflection

• How well do my students know the connections between fractions, proportion and percentage? How might I support them in building these connections?

• Do I understand equivalent fractions and show them on number lines? What might I need to learn more about to support me in doing this with my students?

• How might I teach students to find fractional quantities without using algorithms?

• Do I understand what proportions are? Am I sure? How do I reason about proportions?

• Do I understand the difference between common fractions and decimal fractions? How might I teach about them using connections or do I treat them as different topics?

• How deeply do I understand the difference between denominator and numerator? Do I understand why students need to learn about denominators first? Where can I find out more about this?

• How might I teach fractions using number lines? What can support me in doing this?

• Am I aware of any misconceptions my students have regarding decimal fractions? What might I consider doing to help them overcome these?

• In what ways do we collect data to monitor progress for each student?
D. Using spatial reasoning

Key messages

There are two key ideas in the Geometry strand of the Australian Curriculum: Mathematics that are important for numeracy:

- visualise 2D shapes and 3D objects
- interpret maps and diagrams

Did you know that ‘shapes’ refer to 2-dimensional drawings and figures such as triangles and squares; and ‘objects’ refer to 3-dimensional objects such as a ball, pyramid or cone? It is not possible to have a 3D shape.

In the first idea, the verb visualise is the key; to visualise shapes and objects you need to know about them and be able to see in your mind’s eye what they look like. In order to ‘see’ shapes and objects it helps to think about their features. These features include lines, edges, corners and angles. While the curriculum doesn’t specify ‘knowing the names of shapes and objects’ as a learning goal, we need to share that language in order to teach and learn about them.

The second idea – interpreting maps and diagrams – is concerned with where things are rather than what things are. If we can interpret a map or diagram we can describe how to get from one place on the map to another. This means that students need to learn and use the language of direction, position and location required to do this.

Links to the curriculum

Links to the Australian Curriculum: Mathematics learning area are with the Geometry part of the Measurement and Geometry strand, principally shape and location, and with the numeracy learning continuum ‘Using spatial reasoning’.
### Relevant Australian Curriculum: Mathematics Content Descriptors

<table>
<thead>
<tr>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shape</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Make models of 3D objects and describe key features</td>
<td>Compare the areas of regular and irregular shapes by informal means</td>
<td>Connect 3D objects with their nets and other 2D representations</td>
<td>Construct simple prisms and pyramids</td>
</tr>
<tr>
<td></td>
<td>Compare and describe 2D shapes that result from combining and splitting common shapes</td>
<td>Describe translations, reflections and rotations of 2D shapes</td>
<td></td>
</tr>
<tr>
<td><strong>Location</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Create and interpret simple grid maps to show position and pathways</td>
<td>Use simple scales, legends and directions to interpret information contained in basic maps</td>
<td>Use a grid reference system to describe locations. Describe routes using landmarks and directional language</td>
<td>Introduce the Cartesian coordinate system using all four quadrants</td>
</tr>
<tr>
<td>Identify symmetry in the environment</td>
<td>Create symmetrical patterns, pictures and shapes</td>
<td></td>
<td>Investigate combinations of translations, reflections and rotations</td>
</tr>
</tbody>
</table>

### Extracts from the Australian Curriculum: Mathematics Achievement Standards

- Students make models of 3D objects. They match positions on maps with given information.
- Students compare areas of regular and irregular shapes using informal units. They interpret information contained in maps.
- Students create symmetrical shapes and patterns.
- Students connect 3D objects with their 2D representations. They describe transformations of 2D shapes and identify line and rotational symmetry.
- Students use a grid reference system to locate landmarks.
- Students describe combinations of transformation. They locate an ordered pair in any one of the four quadrants on the Cartesian Plan. They construct simple prisms and pyramids.

### Australian Curriculum Numeracy Learning Continuum

<table>
<thead>
<tr>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Visualise 2D shapes and 3D objects</strong></td>
<td>Visualise, sort, describe and compare the features of objects such as prisms and pyramids in the environment</td>
</tr>
<tr>
<td>Visualise, sort, identify and describe symmetry, shapes and angles in the environment</td>
<td></td>
</tr>
<tr>
<td><strong>Interpret maps and diagrams</strong></td>
<td>Identify and describe routes and locations using grid reference systems and directional language such as north or north east</td>
</tr>
<tr>
<td>Interpret information, locate positions and describe routes on maps and diagrams using simple scales, legends and directional language</td>
<td></td>
</tr>
</tbody>
</table>
Planning

Teachers use the information in the ‘Links to the curriculum’ table to plan by backward mapping from the expected outcomes and make decisions about the explicit teaching and learning experiences/tasks which best meet the needs of the students in their classes. In this key element planning will also include attention to the types of models and equipment which will be used to support visualising shapes and objects.

It will be useful to obtain a copy of the K–2 booklet to both remind yourself of the earlier expectations and to be familiar with the teaching knowledge and strategies suggested in order to provide re-teaching or intervention if necessary for students who are not yet operating within the Years 3–6 curriculum expectations.

The activities in the following ‘Putting it into practice’ section describe the sorts of learning opportunities in using spatial reasoning teachers need to create to develop the required, stated learning, both for numeracy and the mathematics that underpins it. Your students might need more or less reinforcement, depending on where they ‘are at’ in their current learning.

Putting it into practice

By the end of Year 2 students were expected to identify, sort and describe common 2D shapes and 3D objects. This prior learning equips them with the kind of language they need to describe and think about shapes and objects. It has also given them skills for visualisation – a skill that is essential in geometry. By the end of Year 4 they are expected to identify and describe symmetry, shapes beyond their classroom, and angles.

Notice the sequence of the verbs in the numeracy outcome for Level 3: Visualise, sort, identify and describe. Students initially see the shapes and can sort and identify based on their holistic recognition of the shape. They can describe it based on its features which they see when they focus on the shape — its lines, corners (vertices) and how the component parts of the shape fit together; they deconstruct it in their mind. Symmetry and angle are two new features at this level. In K–2 students were able to describe and sort, based on the faces, vertices and edges. They now need to be taught the concepts of symmetry and angle so that they can see these features when they look at shapes; hence, they have a greater range of features that they can use to describe shapes and objects than previously.

Teaching transformations, nets and moving shapes and objects

Transformations for Years 3–6 include translations (slides), reflections (flips), and rotations (turns). These skills are best taught using shapes, and then when children can see what is changing in 2D, you can move to 3D and ask similar questions.

Use the word ‘visualise’ regularly and have students describe what they can ‘see’. Ask them to visualise a square piece of paper. Say: Draw what you see if the paper is folded in half across one of its diagonals? Draw what you see if it is folded in half from half way along one of its sides to halfway along the opposite side. You can see how important language is when you are stating and describing movements. This forces students’ brains to work in different ways. Barrier games are helpful here as students will learn to use explicit descriptions using correct mathematical vocabulary.

Some activities for visualising 2D shapes and 3D objects

• Have students draw a flower with five petals arranged evenly around a circle centre. Say: Turn the flower a quarter turn clockwise; draw your flower now. Turn it another half turn. Draw it now, and so on. Check their final drawing and ask them what they did to get that drawing.

• Bring a box of large chalks to class. Have each student take off their right shoe and colour a pattern on the sole. While they are holding it in their hand and looking at it, get them to draw what the design will look like if they put their shoe on and stamp it on the ground. Have them predict what the image of their shoe will look like: Will it look like the drawing you did? What will be different? Repeat with the other shoe.
• Give students nets of a range of objects, including pyramids and prisms. Have them work in pairs to predict what their object will look like if they fold the net up. Name it and then fold it to check if they are right.

• Have students draw a scribble pattern on a piece of paper. Next glue the paper to a piece of card and cut the card into squares or rectangles to make a type of jigsaw. One student then has to put the jigsaw back together while the other student keeps one piece of the puzzle hidden. The second student has to try to draw what the missing piece looks like based on where the lines of the scribble pattern start and finish in the ‘gap’. Swap and repeat with the second student hiding a different piece.

• Give students a square piece of cardboard. Have them draw four or five straight lines across the cardboard and then cut these lines. They look at and describe the shapes of the cardboard pieces. Then, try and put the ‘jigsaw’ together again to make a square. When students have completed this jigsaw a few times, swap their pieces with another group and see if they can name their pieces and put their jigsaw together.

• Use spare wooden cubes, about six. Have students work in pairs. They glue the cubes together along different faces, then paint the resulting object. The task is to see if they can work out how many faces on the original cubes are painted and how many are not. When they have done this, give their object to another pair and see if they can work out the number of painted and non-painted cubes.

• Give pairs of students a paper circle which they fold in half and in half again. Have them cut off the corner using a design like a jagged line, curved line, or a straight line at an unusual angle. They then have to visualise and predict what the shape will look like in the middle of the circle when they open it up. They share cut circles with other students – handing them to each other folded so they don’t see the shape in the middle – and predict what the shapes will look like.

**Teaching symmetry**

There are two different types of symmetry: line symmetry (sometimes called bi-lateral symmetry) – seen in mirror reflections – and rotational symmetry, which exists when an object or shape rotates (sometimes in partial turns) around a central spot.

Students can find a line of symmetry by placing a mirror on a shape; if a line of symmetry exists then each side of the line will be the same. The shape seen in the mirror will be exactly the same as the shape covered by the mirror. Other activities include folding shapes along a line – if the shape on one side of the line folds exactly onto the shape on the other side, then the folding line is a line of symmetry. Examples:

Lines of symmetry

![Lines of symmetry](image)

Similarly for rotational symmetry: by turning an object or shape around some central point of rotation, the shape or object should completely cover itself. For the pentagon above, there are three lines of symmetry and one centre of rotation where the three lines intersect.

**Teaching angle**

An angle is the geometric terminology for describing what students may earlier have called a corner. It occurs when two lines intersect. By learning about different types of angles students have a greater ability to compare; they can describe angles as less than, equal to or greater than 90°, if they know what 90° looks like. This is best taught as the description of a corner on a square, or the shape formed when a wall joins the ground. With students who are experienced in geometric reasoning (e.g. from Year 5), you might also teach it following instruction on symmetry, as a ‘quarter turn’. They can then compare angles as less than a quarter turn, more than a quarter turn, or equal to a quarter turn, using the quarter turn as their ‘benchmark’. This can also be linked to clock faces, and turning of the hands of the clock provide a meaningful context.
Teaching prisms and pyramids

Prisms have two identical and parallel faces – one at the front and one at the back. The prism is named by the shape of this identical front and back face. A triangular prism, for example, has two triangular faces – one at the front and one at the back. They are identical and they are parallel. These are joined by straight edges. You can use shapes that students are familiar with to teach these objects. You might even set this task as homework, using a Toblerone chocolate bar and then ask students to see if they can find examples of similar objects at home or in their community. You can teach students about square prisms, rectangular prisms (as are most boxes), and circular prisms (otherwise known as cylinders).

Most students are familiar with pyramids and you might use the Great Pyramids of Egypt as an example when teaching this object. These pyramids have a square as their base with each angle of the base joined to the apex at the top. The shape of the base might also be a triangle, rectangle, hexagon, pentagon etc.

Some activities for learning about shapes and objects

• Use barrier games that require students to describe shapes and objects. They can include their new language and understanding of symmetry and angle, prism and pyramid in their descriptions if appropriate. One student describes the shape or object and the other, behind the barrier, attempts to draw it.

• Draw a shape on the whiteboard and have students attempt to describe its features. When all students have finished, have some students read their descriptions out and the rest of the students must judge which is best.

• Have students build prisms (including cubes) and pyramids using pop sticks, straws, toothpicks, blu-tack and other appropriate materials (or make them using apps or online drawing software). They compare these models using a table similar to the one below.

<table>
<thead>
<tr>
<th>Object name</th>
<th>Shapes of faces</th>
<th>Number of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parallel faces</th>
<th>Symmetry</th>
<th>Number of straight edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Using the models they have built, have students compare the top views, side views and base views by looking from different positions and draw what they see.

• Make nets of pyramids and prisms. Get students to fold their nets up so that the objects are sitting on their bases. They then describe the bases and compare each with the faces of the sides of their object. They describe any parallel faces in their prisms.

• Have students use grid paper to make a design by shading six squares in a linear pattern. They then draw a line of symmetry along one side and have their partner reflect the design across the line of symmetry. They then swap and repeat the process. They can check their answers using a mirror.

• Ask students to draw a flower made up of six petals on grid paper. They cut out the flower and then attach it to their paper using a drawing pin. They turn the flower one quarter turn clockwise, half turn clockwise, and three quarter turn clockwise, drawing around their flower at each turn, and describe what they notice.
Some activities for interpreting maps and diagram

In order to interpret maps and diagrams students need to have the necessary language. In K–2 students learned words of position and to follow directions and move somewhere else based on the words they heard such as left and right, and knowledge of the four compass points. In Years 3–6 they develop this understanding further so that they are able to use scale, legends (or keys) and use the four compass points to give directions and locate positions.

• Have students draw a simple map showing their school and other significant places in their community such as the post office, some main roads and the Town Hall. Include the north point. Their task is to describe the position of each building using the compass points to indicate the other buildings. For example, they might say: The school is north of the Post Office and the Town Hall is north east of the school. You might then ask them to draw other buildings on the map given your descriptions.

• Give students a simple map showing places such as parks, school, post office, bank and so on but with no roads on the map. Ask them to draw the roads by connecting the places with straight lines, and then describing the location of one place from another along a road. For example, they might say: The Post office is north of the Town Hall; or the school is west of the Post Office.

• Play ‘Simon says’ with class, giving directions like Simon says rotate one quarter turn on the spot, clockwise. Simon says rotate one three-quarter turn on the spot anticlockwise. Simon says rotate one half turn on the spot clockwise and then one quarter turn clockwise; who can describe their position in terms of how they began? And so on.

• Give students a simple map of the classroom and have them work in pairs to follow directions that you give them verbally and write down answers. Superimpose a grid pattern over the top of the map. Directions might include: On the map of the classroom what is the grid reference of the teacher’s desk? What is the grid reference of the window? What is the grid reference of the whiteboard? What is located at A4? What is located at C5?

Monitoring and assessment

All the described tasks and problems can be used for assessment as well as teaching. Students can be asked individually to find solutions and to draw and write about how they know they are correct. You might also give them a question with a hypothetical student’s response, and have them mark it: Are they right? How do you know?

If students don’t appear to be progressing it is important to have an objective look at your assessment tasks. Are the questions too hard? Do they assess the learning that you planned for? Are there words and phrases in the tasks that students may not be able to read and understand? Do your observations validate the learning or are you seeing different things in the behaviours than you are seeing in written tasks? Have a peer or mentor check the alignment between the tasks and the intended learning.

Past NAPLAN questions can be used to support assessment before, during and after a teaching focus. Teachers might use selected questions related to spatial reasoning from a range of year levels to determine student strengths and misconceptions and target teaching to particular concepts.
Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: Will some maths help here? What maths are we using here?

In **History/Geography** students may learn about navigational routes and explorers’ travels. They also learn about other countries. This learning often involves maps showing positions, distances travelled, key features in the landscape and compass directions. Teachers should use these as opportunities for students to see the value of maps in real-life situations and historical contexts.

In **English** they might use simple grids to determine positions in stage layout and actor positions. They might also come across instructions for camera positions in filming, and talk about camera angles when interpreting visual text. Some texts, especially those involving fantasy, include maps to help readers understand the narrative e.g. map of the *Shire in The Hobbit*. These are opportunities to support students in understanding how to read and interpret scale, legends and positions of places relative to other places.

Questions for reflection

- How well do my students know how to visualise something? How do I know?
- Have I explicitly taught them how to visualise something? How have I modelled this?
- How do I support my students to draw what they see ‘in their mind’s eye’?
- How well are my students able to draw shapes and objects from a written or verbal description? Can they provide descriptions of shapes and objects using mathematical language to describe features? If not, how might I support them to develop this vocabulary?
- Do my students describe an object using technical language appropriate for their age? What words and phrases do I need to model and focus on? What words and phrases might I need to learn?
- How well do my students understand transformations: reflection, rotation and translation; and the practical applications of these for concepts such as symmetry, clockwise and anti-clockwise turns; and drawing parallel faces in prisms? What resources do I have or do I need to access to support this learning?
- Are my students able to recognise and name prisms and pyramids in the environment? How might I support those who need to develop this understanding?
- How do I plan experiences which enable my students to follow directions given in language that includes compass points, scale, turns, and left and right to go somewhere? What tasks will enable them to give simple directions using the same sort of language to explain to someone how to get somewhere?
- In what ways do we collect data to monitor progress for each student?
E. Interpreting statistical information

Key messages

There are two key ideas in the Statistics and Probability strand of the Australian Curriculum: Mathematics that are important for numeracy:

- interpret data displays
- interpret chance events.

Data that is collected, organised and displayed by someone else is called secondary data; if we collect, organise and display it ourselves it is called primary data.

In the first idea, interpret data displays, the verb interpret is the key. Interpreting data is the culmination of the collecting, organising and displaying data – it is the purpose for engaging in these activities. Technology can be increasingly programmed to collect, organise and display or represent data, but it is difficult to program it to effectively interpret the data. You cannot learn to interpret data by engaging only in the skills of collecting, organising and displaying it. People don’t learn what graphs mean by drawing them, for example. It is helpful to engage in some data collection and organising activity in order to understand its purpose and context it. However, the focus is on the interpreting.

The second idea – interpreting chance events – is concerned with interpreting information about the likelihood of something happening. Words of chance occur in the homes of most children. They include words and phrases like maybe, perhaps, might, could, won’t, can’t, impossible, and even Australian colloquialisms such as pigs might fly, in your dreams, sure thing, fifty-fifty, and so on. Most children grow up hearing these words and get a sense about likelihood by hearing them used in different situations and circumstances.

Students need to be taught what phrases like little chance or big chance mean so that they can make informed decisions when taking risks. The complexity and depth of understanding increases as we have greater experience with risk in our lives.

Links to the curriculum

Links with the Australian Curriculum are with the content areas from the Data Representation and Interpretation strand of the Mathematics curriculum, and with the Interpreting statistical information element of the numeracy learning continuum.
### Relevant Australian Curriculum: Mathematics Content Descriptors

<table>
<thead>
<tr>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data representations and interpretation</strong></td>
<td>Select and trial methods for data collection, including survey questions and recording sheets</td>
<td>Pose questions and collect categorical and numerical data by observation or survey</td>
<td>Interpret and compare a range of data displays, including side-by-side column graphs for two categorical variables</td>
</tr>
<tr>
<td>Identify questions or issues for categorical variables. Identify data sources and plan methods of data collection and recording. Collect data, organise into categories and create displays using lists, tables, picture graphs and simple column graphs, with and without the use of digital technologies. Interpret and compare data displays.</td>
<td>Construct suitable data displays with and without the use of digital technologies, from given or collected data. Include tables, column graphs and picture graphs when one picture can represent many data values. Evaluate the effectiveness of different displays in illustrating data features including variability.</td>
<td>Describe and interpret different data sets in context.</td>
<td>Interpret secondary data presented in digital media and elsewhere.</td>
</tr>
<tr>
<td><strong>Chance</strong></td>
<td>Describe possible everyday events and order their chances of occurring. Identify everyday events where one cannot happen if the other happens. Identify events where the chance of one will not be affected by the occurrence of the other.</td>
<td>List outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions. Recognise that probabilities range from 0 to 1.</td>
<td>Describe probabilities using fractions, decimals and percentages.</td>
</tr>
<tr>
<td>Conduct chance experiments, identify and describe possible outcomes and recognise variation in results.</td>
<td></td>
<td></td>
<td>Conduct chance experiments with both small and large numbers of trials using appropriate digital technologies.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Compare observed frequencies across experiments with expected frequencies.</td>
</tr>
</tbody>
</table>

### Extracts from the Australian Curriculum: Mathematics Achievement Standards

| Students interpret and compare data displays. They carry out simple data investigations for categorical variables. They conduct chance experiments and list possible outcomes. | Students describe different methods for data collection and representation, and evaluate their effectiveness. The construct data displays from given or collected data. They list the probabilities of everyday events. They identify dependent and independent events. | Students compare and interpret different data sets. They pose questions to gather data and construct data displays appropriate for the data. They list outcomes of chance experiments with equally likely outcomes and assign probabilities between 0 and 1. | Students interpret and compare a variety of data displays including those displays for two categorical variables. They evaluate secondary data displayed in the media. They list and communicate probabilities using simple fractions, decimals and percentages. |
**Planning**

Teachers use the information in the ‘Links to the curriculum’ table to plan by backward mapping from the expected outcomes in relation to interpreting statistical information. In particular, they need to understand precisely what each content descriptor means. Teachers need to ask: What do I need to do to enable them to do and know these things? This is the pedagogy question and requires attention to planning for the explicit teacher modelling and teaching as well as the tasks which will challenge and consolidate student understanding of key ideas. In this key element attention to planning for explicit teaching of the vocabulary of statistics and probability is also important.

The activities in the following ‘Putting it into practice’ section describe the sorts of learning opportunities for interpreting statistical information that teachers need to create to develop the required, stated learning, both for numeracy and the mathematics that underpins it. Your students might need more or less reinforcement, depending on where they ‘are at’ in their current learning.

### Australian Curriculum Numeracy Learning Continuum

<table>
<thead>
<tr>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interpret data displays</strong></td>
<td>Collect, compare, describe and interpret data in 2-way tables, double column graphs and sector graphs, including from digital media</td>
</tr>
<tr>
<td>Collect, record and display data as tables, diagrams, picture graphs and column graphs</td>
<td></td>
</tr>
<tr>
<td><strong>Interpret chance events</strong></td>
<td>Describe chance events and compare observed outcomes with predictions using numerical representations such as 75% chance of rain or 50/50 chance of snow</td>
</tr>
<tr>
<td>Describe possible outcomes from chance experiments using informal chance language and recognising variations in results</td>
<td></td>
</tr>
</tbody>
</table>

### Putting it into practice

In the K–2 years, students learned to collect data based on simple questions, usually with one or two categories. The question might have been: What is your favourite colour: red, blue, green, or yellow? Or it may have just been a yes/no question such as: Do you like swimming? In Year 3 these types of questions that result in one piece of information per student, can also be used. However; students now need to be suggesting the questions themselves. When they collect data to answer their own questions about things in which they are interested, the collecting has a purpose for them. They might still display this data using one picture or dot for each student but by the end of Year 4 they will be able to group their data and use one picture or dot to represent a small number of students. If their data results in a numerical total for each category it makes sense for students to be drawing column graphs; the height of the row of dots or pictures can be found by joining the data points to make a column.
Some activities for interpreting data displays

- Ask students to think of a question that they want to find information about. They think about the possible answers they might get to their question and use these as categories in the table they use to collect their data. For example, if they are interested in what the favourite foods of their peers might be, they might construct a table such as:

<table>
<thead>
<tr>
<th>Favourite food</th>
<th>Tally</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>pizza</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fruit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eggs</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Students should display this data in a graph, deciding whether to use one dot/picture for each student asked or, as they get older, to group the data and use one dot/picture to represent a number of students. Students should write questions about their graph that other students can answer by interpreting their graph. For example: Which is the favourite food for students in this class? Do students prefer eggs or fruit? What is the least favourite food for students in this class?

- Show students a display/graph of some information (such as the one below) and ask them to consider how the data was collected: What questions might have been asked? What type of table or recording sheet was used to collect and organise the information. Who might have been asked?

**Pocket Money**

```
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Week</td>
<td>#</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>1</td>
<td>#</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>#</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>#</td>
<td>#</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>#</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

- Have students work in pairs to choose a question that they want a yes/no answer for. They ask all students in their class and carefully record their answers in a table with a tally, such as:

<table>
<thead>
<tr>
<th>Like skateboarding</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Don’t like skateboarding</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask students to display their data in a chart that uses a symbol/drawing to represent a number of students. The symbol has to be easy to reproduce or something they can find on the web, cut and paste, and repeat. For example:

```
<table>
<thead>
<tr>
<th>Like skateboarding</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Don’t like skateboarding</td>
<td></td>
</tr>
</tbody>
</table>
```

**key**  ■ = 2 students

They can be asked to consider how they might show one student or three students; the class might discuss this at length.

- Give students two-category data displayed in a side-by-side column graph and ask them to write a paragraph about what the data indicates. They must then write questions about the data for their partner to answer.

- For displays (such as the one below), have students enter their data on a spread sheet and plot using both a column, and then a bar graph. Ask them: Which graph is the best way of showing the data? Why?

**Students in Year 6**

```
<table>
<thead>
<tr>
<th></th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>boys</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>girls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

**Number of Students**

```
<table>
<thead>
<tr>
<th></th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>boys</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>girls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```
Interpreting statistical information

Some activities for interpreting chance events

It is important for teachers to know that chance isn’t about probability. In fact, it’s the other way around; probability is about chance. Probability is about measuring chance or measuring likelihood. Most people in their everyday lives aren’t concerned with measuring the likelihood of something happening or not. They are satisfied to have a general sense about likelihood – this is demonstrated in the language we use when talking about chance.

In the early years students learned phrases such as might happen, and might not happen, certain and impossible. They were able to give examples of events that were certain and impossible and they learned that some events were more possible (or likely) than others. In Year 3 they are able to describe events using the language of likelihood based on their experiences in the classroom (such as throwing dice, throwing coins) and events at home. They should use their learning in collecting and displaying data to organise and manage results obtained through chance activities; tossing a coin 20 times for example, they can record the result of each experiment (toss). Having the data will help them consider possibilities and likelihood resulting from these events.

Students need to learn that some events are independent of each other. Tossing a coin and then tossing it again are independent events since the result of the second is not influenced by the result of the first. On the other hand, some events are dependent on each other; i.e. the result of one experiment can sometimes affect the result of the next experiment. For example, if you draw out a card from a standard pack and get the queen of hearts and don’t put it back, you can’t get the queen of hearts on the next draw; the result of your second draw depends on the result of your first.

You should teach students that we can use numbers as a measure of chance; that ‘50% chance’ means it is equally likely to happen or not happen. Students should learn that most of numbers used to indicate likelihood (e.g. ‘there is a 30% chance of rain tomorrow’) come from data gathered over many, many years (or experiments). Students learn that by tossing a coin 10 times it is unlikely that they will get exactly five heads and five tails. If someone else in the class tosses a coin 10 times they are very likely to get different results – this variation in results is normal and should be expected.

- Have students work in pairs to match the word that best describes the chance of the event occurring: impossible, unlikely, likely, more likely, certain

<table>
<thead>
<tr>
<th>Event</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>I will have sand for dinner</td>
<td></td>
</tr>
<tr>
<td>My big brother will pick me up after school</td>
<td></td>
</tr>
<tr>
<td>My hair will turn green overnight</td>
<td></td>
</tr>
</tbody>
</table>

- Have students work in pairs to toss a die 30 times and record their results. When they are finished ask them to consider the following question:

Freda was tossing a coin 30 times. She made the following table to show her results:

<table>
<thead>
<tr>
<th>Head</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tail</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>18</td>
</tr>
</tbody>
</table>

How many more times did Freda throw a tail than a head? Is her next throw more likely to be a head or a tail? Why? Have them look at their own results and think about what they might expect to get on their next throw, and give reasons.
• Have students get into pairs, each with a coin. They should toss their coin 100 times and predict how many heads and tails they will get. Record the results for each pair in the class. Discuss the fact that there are some with more than fifty heads and some with less than fifty heads, so in the long run they are more likely to have about 50 of each.

• Have students make their own spinners with ten sections of different colour and work out the chance of their arrow landing on each colour. They should write their probabilities using fractions or percentages.

When students use spinners, some believe that if there appears to be more of one colour than another, there is more chance that their spinner will land on it. For example, with the cards below when the spinners are placed in the centre, some might believe the spinner is more likely to land on the grey in spinner B, because they perceive there is more grey than in spinner A:

![Spinners A and B]

• Give students a set of spinners with the same amount of shading (albeit in different sections) and have them arrange them in order from those where the spinner is more or less likely to land on the shading. Then have students compare their results with other students and discuss possible reasons.

Monitoring and assessment

Assessing the interpretation of data in Years 3–6 involves giving students lots of practice with different types of displays, including tables and charts. Students need to be explicitly taught some strategies for doing this in step-by-step and holistic ways, since a data display can be a very complex text to read.

The tasks outlined can all be used for assessment as well as for teaching and learning. Even though you have used some contexts for teaching you can always adjust the tasks for assessment by changing the context. The focus should be on students interpreting the data, rather than collecting and organising it.

A good source of assessment questions for children in Years 3–6 is some of the questions in past Year 3 and 5 NAPLAN Numeracy test papers (it is also important to expose Year 6 students to the types of questions they will meet in Year 7 through using questions from the Year 7 test). The questions generally align closely with the definition of numeracy and:

• provide no hints about the mathematics children should choose to use
• are for the most part written in contexts so that children have to understand these before choosing the mathematics to use.

Teachers might for example use the IMPROVE website to select past NAPLAN questions focussing on interpreting statistical information and create and online quiz for students. The students can immediately access their results and set goals for improvement. Teachers can use the results to see patterns in student need and target teaching to address particular areas of strength or weakness. Likewise the NAPLAN Toolkit can provide teachers with teaching strategies based on individual student achievement in past NAPLAN testing.
Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: Will some maths help here? What maths are we using here?

In Science lessons when students are learning inquiry skills they might use contexts that generate data over time e.g. growth of plants. You would encourage them to collect their data using tally points or informal measurements, and record their observations in tables or spread sheets.

In History students are learning to distinguish between past, present and future (sorting and ordering time and dates), and they are learning to pose questions about the past from a range of sources and exploring these sources (collecting data).

In English you might encourage students to support their arguments with data and graphs when composing a persuasive piece of writing.

Questions for reflection

- How well do my students understand that different charts are used to display data for different purposes and audiences? That they should decide how to display the data based on the question it is answering? What do I need to learn/know to be able to support them in developing these understandings?
- How well do I understand what variation in data is about? What might I need to learn help me in teaching this?
- How well do I know how to read graphs and interpret them?
- How well do I understand the words of chance like observation, likelihood, event, experiment? How confident am I to teach these words? What learning might build my own confidence in these ideas?
- How do I support my students to understand the difference between dependent and independent events?
- What is my understanding of what risk is about? Do I understand that while risk can be managed and minimised, it can’t be controlled?
- In what ways do we collect data to monitor progress for each student?
F. Using measurement

Key messages

There are two key ideas in the Measurement and Geometry strand of the Australian Curriculum: Mathematics that are essential for numeracy:

- estimate and measure with metric units
- operate with clocks, calendars and timetables

Both of these ideas are about measurement rather than geometry.

Estimate and measure are the two verbs used in the first idea and their order is important; there is no point estimating after you have measured. Sometimes an estimation is all that is required in the context in which you are working. If you don’t need to be very accurate and you can tolerate some error, an estimation might be sufficient. For example, if you want to buy a new wardrobe for your bedroom, you might use a hand span to decide how wide it will need to be to fit all your clothes in or how narrow it needs to be to fit through your bedroom door.

As with calculation, it is impossible to estimate without deep knowledge of what you are estimating. If you don’t have a deep understanding of the attributes length, area, volume, capacity and the units used to measure them – in this case, metric units – you will not be able to estimate their measurements. To measure requires some deep understandings about:

What needs to be measured (attributes).

- **Units** of measurements (so you can estimate and choose an appropriate tool to measure with).
- **Estimation**, using understanding of attributes and units.
- **Measuring**, using direct (doing the measuring yourself) and indirect (using a combination of measures or measures given to you) methods.

Although time is an attribute, it is singled out because it is slightly different from the other attributes of length, area, mass and volume/capacity and angle; we can’t see/heft it – we experience its passing.

Links to the curriculum

Links with the Australian Curriculum are with the content areas from the Geometry and Measurement strand of the Mathematics curriculum, and with the Using Measurement element of the numeracy learning continuum.
### Relevant Australian Curriculum: Mathematics Content Descriptors

<table>
<thead>
<tr>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Using units of measurement</strong></td>
<td>Use scaled instruments to measure and compare lengths, masses, capacities and temperatures</td>
<td>Choose appropriate units of measurement for length, area, volume, capacity and mass</td>
<td>Connect decimal representations to the metric system</td>
</tr>
<tr>
<td>Measure, order and compare objects using familiar metric units of length, mass and capacity</td>
<td>Compare objects using familiar metric units of area and volume</td>
<td>Calculate perimeter and area of rectangles using familiar metric units</td>
<td>Convert between common metric units of length, mass, and capacity</td>
</tr>
<tr>
<td><strong>Time</strong></td>
<td>Convert between units of time</td>
<td>Compare 12 and 24-hour time systems and convert between them</td>
<td>Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>Tell time to the minute and investigate the relationship between units of time</td>
<td>Use am and pm notation and solve simple time problems</td>
<td>Interpret and use timetables</td>
<td>Connect volume and capacity and their units of measurement</td>
</tr>
</tbody>
</table>

### Extracts from the Australian Curriculum: Mathematics Achievement Standards

- **Students use metric units for length, mass and capacity.** They tell time to the nearest minute.
- **Students compare areas of regular and irregular shapes using informal units.** They use scaled instruments to measure temperatures, lengths, shapes and objects. They classify angles in relation to a right angle. They convert between units of time. They solve problems involving time duration.
- **Students use appropriate units of measurement for length, area, volume, capacity and mass, and calculate perimeter and area of rectangles.** They measure and construct different angles. They convert between 12 and 24-hour time.
- **Students connect decimal representations to the metric system and choose appropriate units of measurement to perform a calculation.** They make connections between capacity and volume. They solve problems involving length and area. They solve problems involving the properties of angles. They construct simple prisms and pyramids. They interpret timetables.
Using measurement

Australian Curriculum Numeracy Learning Continuum

<table>
<thead>
<tr>
<th></th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimate and measure with metric units</strong></td>
<td>Estimate, measure and compare the length, temperature, volume, capacity and mass of everyday objects using metric units and scaled instruments</td>
<td>Choose and use appropriate metric units for length, area, volume, capacity and mass to solve everyday problems</td>
</tr>
<tr>
<td><strong>Operate with clocks, calendars and timetables</strong></td>
<td>Read digital and analogue clocks to the minute, convert between hours and minutes, use ‘am’ and ‘pm’, and use calendars to locate and compare time events</td>
<td>Convert between 12 and 24-hour systems to solve time problems, interpret and use timetables from print and digital sources</td>
</tr>
</tbody>
</table>

Planning

Teachers use the information in the ‘Links to the curriculum’ table to plan by backward mapping from the expected curriculum outcomes for using measurement.

To plan effectively, teachers also need to have some understanding of what their students already know, understand and can do through pre-assessments. It will be useful to obtain a copy of the K–2 booklet to both remind yourself of these earlier expectations and to be familiar with the teaching knowledge and strategies suggested in order to provide re-teaching or intervention if necessary. Planning for this key element also requires attention to the measuring tools which students will be exposed and expected to use across the curriculum. For example, are there opportunities to use stopwatches or smart phone apps for measuring in Physical Education, how might particular tools be used to measure in science?

Activities in the following ‘Putting it into practice’ section describe the sorts of learning opportunities for using measurement that teachers need to create to develop the required, stated learning, both for numeracy and the mathematics that underpins it.

Your students might need more or less reinforcement, depending on where they ‘are at’ in their current learning.

Putting it into practice

1. Understanding attributes

Part of being numerate in measurement is understanding what needs to be measured in a context. It is therefore important that students understand measurement attributes. An attribute is a quality that an object has. It might have length or area, mass or volume/capacity, or angle, and often it will have more than one of these. If we want to measure something we are interested in how much of a particular attribute it has.

2. Understanding units

It is impossible to estimate measures without understanding the units that we use for measuring. Although the element of the numeracy learning continuum is ‘Estimate and measure with metric units’ young children estimate and measure with non-standard units before they can do these things with metric units. In the K–2 booklet many activities focused on the understanding of what measuring is; and non-standard units were used for this purpose.

Measuring requires us to work out how many repeats of the unit we use are needed to match the thing to be measured. This is essentially what measuring is about. So the understandings for measurement, using units include:

a. Some objects work better than others as units to measure with (due to gaps and overlaps).
b. The bigger the unit chosen to measure with, the smaller the number of repeats.

c. For comparison you need to use the same unit to measure all objects.

d. The purpose for measuring tells us which unit will give the most accuracy (and how much care is needed in measuring).

e. Standard units are no more correct than non-standard units; it depends on the context.

As students move through the years of schooling they are able to work with standard units as a more accurate way of communicating about measures. It is helpful if we choose to measure with a unit that relates well to the attribute we are measuring with. For example, we wouldn’t use a ruler to measure mass, even though a ruler would be a good unit to use if we were measuring length.

Understanding the units we use is helped if we can visualise them. Students need to be able to ‘see’ metric units and use these mental images as benchmarks. For example, if we can ‘see’ a square metre, we can look at a floor rug and estimate how many square metres could fit onto the rug — hence our estimation is based on the mental image we have of the square metre, our benchmark.

3. Estimating measures

The link between understanding attributes and units, and estimating becomes clear when you consider what is needed in order to estimate. Clearly you can’t estimate how much of an attribute something has if you don’t understand the attribute or the units used to measure it. For example, how can you estimate the area of a carpet if you don’t understand what area is and if you don’t have a mental picture of a unit used to measure it, such as a square metre?

Estimation is an ‘informed guess’. It is making an approximation based on the information available to you; what you can see, heft, or experience (in the case of time). Estimation is helpful when it is difficult to measure something directly. Estimation depends on purpose and audience. It is these two criteria that help us decide whether ‘near enough is good enough’, are we confident that our estimate is good enough in the circumstances or should we calculate with greater precision?

4. Measuring (direct and indirect)

Direct measuring requires us to work out how many repeats of the unit we use are needed to match the thing to be measured, and to do this ourselves rather than using a formula or measurements that someone else has made. Initial measuring tasks involve counting units and we need to be careful that our students learn that measuring and counting are not the same thing; although the understanding of counting is foundational to direct measurement. If you deeply understand counting you will know that a piece of a ruler is an appropriate tool to measure the length of a pencil for example — you don’t need the whole ruler and you don’t need to start at ‘0’.

The smaller the unit used to measure with, the greater the accuracy. In using a cup there will always be some spillage or error since you are only using your eye to ‘fill’ the cup. In using a 30 cm ruler marked with millimetres you are more likely to reduce the measurement error when measuring the length of a pencil than if you used a whiteboard ruler marked with centimetres.

Students should learn to measure using ‘between’ or ‘to the nearest’ statements.(x)

They wouldn’t say ‘this jug holds six cups of water’ but rather ‘this cup holds between five and seven cups of water’ or ‘this jug holds six cups of water to the nearest cup’.

Students need to learn what the appropriate measuring tool is to measure each attribute and to use the tool in ways that reduce measurement error. They should complete tables such as:

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Suitable units</th>
<th>Suitable measuring tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of pencil</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height of door</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of an egg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume of a box</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacity of a cup</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area of a book cover</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height of a tree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area of a rug</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Width of a chair</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(x) Education Department of Western Australia, 2004
Students should have many opportunities to talk about which unit or measuring tool is best for measuring each of these attributes and why or why not. If young children have not fully grasped these concepts they need to be revisited in Year 3. You might assess understanding of these concepts using diagnostic tasks to provide a sense of how deeply you need to go into these concepts again. Don’t assume that all students mastered them in the early years.

Indirect measuring also requires us to work out how many repeats of the unit are needed to match the thing to be measured, but it is applied when the object or shape is large or complex. When finding the area of a large park for example, it would be too laborious or time-consuming to use a square metre tool and lay it out over the park repeatedly to find how many square metres of ground surface in the park. Instead, we might use a formula made up of a number of different measures (e.g. area = length x width).

In learning to measure young children are not expected to use formulae. They will, however, use indirect measuring by combining direct measures or to compare and order attributes. For example, they might compare the lengths of a range of objects they can move by placing them alongside each other. If the objects are too large or fixed in position they can compare by using a third, movable object, as a ‘go-between’. For example, students can compare the heights of doors by using a broom as their ‘matching unit’. Hold it against one door and say: ‘this door is a lot higher than the broom’; then move the broom against another door and say: ‘this door is about the same height as the broom, so the first door must be higher than the second door’. Students might then move to a third door and repeat the process. This will result in them being able to compare the heights of the three doors.

Some activities for estimating and measuring with metric units

- Give students a task requiring them to determine how much ice cream they will need to buy for the class if each student has a single scoop measuring about 200 grams. How can they buy it in the supermarket? They could bring grocery junk mail to class or go online to the local supermarket. How many containers full will they need to buy? Extend this by asking them to calculate the total cost.

- Provide a recipe that requires a number of ‘cups’ of flour. Ask: Will the size of the cups always be the same? How do we know? Won’t everyone’s Mum or Dad use a different cup? How could we make sure it’s always the same? How much does a standard cup hold?

- Ask students to work together to make a cake for their class. The recipe calls for 6 kg of flour but their scales only go up to 800 grams. Ask: How many ways can you measure the flour?

- Pair students to work together to measure the height of the window in their room; they know that the window goes to the ceiling which is 2.5 metres high and the window sill is 70 cm from the floor. Have each pair explain and write what they would do to calculate the answer.

- Provide a thermometer for students to examine. Ask: Where does the liquid start – at the ‘bowl’ or the ‘0’ mark? Have them research and then write instructions for someone else about how to read a thermometer to tell the temperature.

- Give students the task of determining how much juice to purchase for a class picnic. They will need to consider how much each person will drink and questions such as: What ‘cup’ will we use to decide this and how much does it hold? How many cups of juice will we need? How large is the container that we can buy juice in? How much does it hold? How many will we need? They can work as a class or in pairs and present their work as a report showing their answers clearly and supported by reasons.
• Have students research the difference between area of a rectangle and perimeter of a rectangle. They should develop a five-minute PowerPoint presentation, using the ‘border’ and ‘shade’ functions to clearly define and indicate the difference, and different types of rectangles to illustrate how these amounts can change. Their presentation should address questions such as: What happens to the perimeter and the area if the rectangle is doubled in length? Halved in length? What happens if it is twice as long and half as wide?

• Give pairs of students two different measuring instruments to measure how much a bucket holds; give them one bucket, and one student a litre container and one a half litre container. Have them measure how many of their containers are needed to full the bucket and discuss differences and why these might have occurred.

• Have students work in pairs, each taking a turn to throw a beanbag. Then have one measure the distance thrown using a strip of paper 90 cm long and another measure the distance thrown using a 30 cm ruler. Ask: What would we expect to find? Is it different? Why do we think so? Is it more accurate to measure using a 30 cm ruler? Why do you think so?

• Have students bring in a box each. They should work as a class to put the boxes in order from ‘holds more’ to ‘holds less’.

• Give students a picture on a piece of A4 paper. Their job is to measure around the edge of the picture to determine how big their frame for the picture should be. Compare every student’s measure for the perimeter of the picture and discuss differences.

• Have students use newspaper and tape/glue to make a square metre.

• Bring a fridge box (or other large box) to school and asks students to close their eyes and visualise a cubic metre. They then have to look at the box and estimate the volume of the box. Is it bigger than a cubic metre? Is it smaller? Is it the same?

• Give students a composite shape made up of straight lines but not rectangles. Ask them: If you were a caterpillar which of these shapes would you prefer to eat if they were patches of grass and why? Give them a transparent grid to work out the areas of their shapes. Discuss each student’s reasoning and either give it the ‘thumbs up’ or ‘thumbs down’.

• Pose questions such as: Jemma bought a floor rug with an area of 1.75 square metres. Will it fit on the floor of her bedroom which is 1.35 m long and 1.45 m wide? If not, how much bigger is the rug, and if so, how much smaller is the rug?

• Pose problems such as: Two students, Greg and Paul, stood back to back. Paul is a quarter of a metre taller than Greg who is 1.05 metres. How tall is Paul?

• And: Alain made a cake the weighed 456 grams. Steve’s cake weighs 0.47 kilograms. Whose cake weighs more and by how much?

• Draw a thermometer that is showing a temperature of 37°C. Get students to indicate gradations every five degrees.

• Have students draw an analogue clock and also a 24-hour digital clock, both showing five past six in the evening.

• Pose problems such as: Four students were arguing about the length of shoes that most Year 6 students would wear. They each made an estimation: Craig said 4 cm, Eli said 24 cm, Nat said 75 cm and Ben said 100 cm. Students visualise each of these lengths and determine which of the boys is more likely to be correct.

• Present a problem such as: Four toddlers were weighed on the same day at the clinic. Simon weighed 6.5 kg, Sarah weighed 6450 g, Meredith weighed 6.04 kg, and Ozzie weighed 6082 grams. Which toddler has the greatest mass?

• And: Franci makes a punch for her party. She uses at least one full bottle of each of the following bottles of juice: orange juice… one bottle contains 500 ml, lemon juice…one bottle contains 200 ml, apple juice…one bottle contains 300 ml… and guava juice…one bottle contains 600 ml. How many full bottles of juice will Franci use to make two litres of punch?
Some activities for operating with clocks, calendars and timetables

Young children often find it difficult to learn about time because they can’t see it. They can, however, experience the passing of time and can learn to accurately estimate it by experiencing how it feels and using familiar personal lengths of time as ‘benchmarks’ e.g. the time it takes to watch a TV program or to drive to school.

As with other attributes, time is measured by counting units. Students need to understand the units of time just as they need to understand units they can see or heft. Students often find it difficult to comprehend that on an analogue clock the hour hand is constantly moving around, albeit slower than the minute hand. Have a digital clock on the wall next to the analogue clock so you can frequently draw attention to the different ways the time is shown on both.

- Give students a list of times in writing: half past two, quarter to one, ten past seven, half past five, 14:21, and three fifteen pm. Have them write the times using digits and also on the clock faces – analogue and digital. Next get them to put the times in order from earliest to latest.
- Give students tables to complete with different times, clocks and words:

<table>
<thead>
<tr>
<th>Time in words</th>
<th>Time on digital clock</th>
<th>Time on analogue clock</th>
<th>Different words for the same time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter to 5 in the morning</td>
<td>04:45</td>
<td>Draw clock face</td>
<td>Four forty-five Quarter to five</td>
</tr>
</tbody>
</table>

- Have students mark these in pairs and correct each other’s work when needed.
- Give pairs of students a calendar and ask them to draw a circle around the date of their birthdays. Put their name next to the date. Have them work together to write five questions about these dates and give to another pair to answer.
- Give students a list of times taken for twenty people to complete a fun run, written in digital format e.g. 20.07 minutes. Have students work together in pairs to put them in order to find the first ten placeholders.
- Download sections of train and bus timetables for your local area and give them to students, having them work in pairs to write questions about time taken and finishing times e.g.

<table>
<thead>
<tr>
<th>Time</th>
<th>Station</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:45</td>
<td>Esplanade Station</td>
</tr>
<tr>
<td>9:46</td>
<td>Canning Bridge Station</td>
</tr>
<tr>
<td>9:52</td>
<td>Murdoch Station</td>
</tr>
<tr>
<td>9:57</td>
<td>Bull Creek Station</td>
</tr>
<tr>
<td>10:03</td>
<td>Cockburn Central Station</td>
</tr>
<tr>
<td>10:10</td>
<td>Kwinana Station</td>
</tr>
<tr>
<td>10:13</td>
<td>Wellard Station</td>
</tr>
<tr>
<td>10:21</td>
<td>Warnbro Station</td>
</tr>
<tr>
<td>10:35</td>
<td>Mandurah Station</td>
</tr>
</tbody>
</table>

- a. How long does it take the bus to go from Murdoch to Warnbro?
- b. Is it faster to go from Bull Creek to Wellard than from Canning Bridge to Kwinana? By how much?
- c. The express train from Esplanade to Mandurah takes 19 minutes. How long will it take to go from Esplanade to Mandurah stopping at all stations? How much time should you save by catching the express train?
- The sun rises at 6.34 am and sets at 7.13 pm. How many hours of sunlight is this?
- If the sun sets at 19:23 and we have 15 hours of sunlight, what time on the 24-hour clock does it rise?
Invite older students to plan an overseas trip, using a 24-hour clock to schedule their departure and arrival times and calculating the time taken to travel between locations and the differences in time zones.

Monitoring and assessment

All of the tasks and learning activities listed above provide opportunities to assess students’ skills and understandings in using measurement. Observations of how students go about solving problems and careful attention to their explanations and reasoning provide teachers with insights into where students are in their learning and inform future planning.

In order to monitor the learning against what students are expected to learn, teachers need to refer to the intended learning in the Australian Curriculum. Assessment must focus on assessing in relation to this intended learning rather than what is currently being taught – the aim is to chart students’ progress towards desired goals.

A good source of assessment questions for children in Years 3–6 is some of the questions in past Year 3 and 5 NAPLAN Numeracy test papers (it is also important to expose Year 6 students to the types of questions they will meet in Year 7 through using questions from the Year 7 test). The questions generally align closely with the definition of numeracy and:

- provide no hints about the mathematics children should choose to use
- are for the most part written in contexts so that children have to understand these before choosing the mathematics to use.

Teachers may for example select questions focusing on measurement from across a range of year levels and print them out onto cards. These cards can then be distribute to groups of students to discuss and then share their thinking back to the whole class. Teachers can then make observations about the strategies students use and understand the misconceptions some students may have. This informs future planning for targeted teaching.

Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: Will some maths help here? What maths are we using here?

In Science lessons when students are learning inquiry skills they might use contexts that generate data over time e.g. growth of plants. They would use metric units in measuring how much a plant has grown over night, or measure different amounts of sugar and water solutions to compare dissolving rates. You would involve them in the decisions about which tool to use for measuring to obtain the level of accuracy required, and how to graph their measurements to best display their data for purpose and audience. You would point out to them that ‘we are using some maths here’, or you might use such a lesson to teach the concepts with all students using the same unit so they can compare whose plant has grown the most.

In History, students can apply their knowledge of time by placing historical events in order by date from most recent to least recent. They can also use their measurement knowledge in Geography by determining and comparing the length of voyages and treks across continents when learning about explorers.

In English, students might measure surfaces on stage to determine the area needed for a dramatic production; encourage them to decide which units of measurement to use and which tools to use for the measurements. You could discuss: Are estimates enough?
Questions for reflection

- How do my students demonstrate that they can visualise standard metric units of length, area and volume?
- What opportunities do I provide to enable students to estimate the number of metric units in a shape or object by looking at the shape or object?
- How well do my students understand what the attributes of perimeter and area are? How do I know? How do my students compare how much of an attribute a shape or object has and use the appropriate language to explain why?
- Do my students know what is and what isn’t an appropriate unit to measure an attribute, and can they explain why?
- Do my students understand the difference between area and perimeter? If not, how might I support them to build this understanding?
- Do my students estimate lengths, areas, perimeters and volumes by visualising metric units and comparing?
- How well do my students tell the time on analogue and digital clocks? If some students are finding this difficult, how might I support them in developing this important skill?
- In what ways do we collect data to monitor progress for each student?
References and Further Reading


Perso, T. 2003, *Everything you want to know about Algebra Outcomes for your class, K-9*, Mathematical Association of W.A. Perth


APPENDIX 1: USEFUL RESOURCES

The following are examples of resources available to support numeracy development.

Key national and state documents

- Supporting Literacy and Numeracy Success [www.education.tas.gov.au/documentcentre/Documents/Supporting-Literacy-and-Numeracy-Success.pdf](staff only)
- Good Teaching: Differentiated Classroom Practice [www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Differentiated-Classroom-Practice-Learning-for-All.pdf](staff only)

Whole school approaches

- Department of Education Improvement Plan – Improving Student Achievement through a Whole-School Approach [https://www.education.tas.gov.au/documentcentre/Documents/Improving-Student-Achievement-Through-a-Whole-School-Approach.pdf](staff only)
Numeracy assessment

**Assessment for Common Misunderstandings**  
(Victorian Department of Education)  
One on one assessment of key ideas in number (development of other strands under way).  
Freely available from this site and through Scootle. Highly diagnostic with teacher advice rubrics on where to go next to target teaching.

**Victorian Early Years Numeracy Interview**  
  
Available online to Victorian Education Dept. Teachers – not DoE (further information)  
One on one assessment of key ideas in number, space and measurement.  
Highly diagnostic and indicates growth points for student understanding. Teacher advice on where to go next for teaching focus.  
Useful for Years F-3 and for use with students who may have misconceptions in years 3–6.

**Junior Assessment of Mathematics (JAM)**  
(New Zealand)  
Freely available.  
Focuses on understanding how students think and reason about each strand of mathematics.  
One on one interview.

**IMPROVE**  
- www.improve.edu.au  
Access past NAPLAN questions and develop quizzes and tests for formative assessment. Free to teachers.  
Logging in: Teachers in Tasmanian Government schools log in using their DoE username and password.

For an overall analysis of a range of tools see this report on diagnostic assessment commissioned by the Australian Government which gives an analysis of key tools for literacy and numeracy  
Teaching practices

**AITSL:** information and videos on quality numeracy practices.


**TCH: The Teaching Channel**

An excellent collection of videos demonstrating effective classroom strategies for numeracy teaching. It has been developed to support the US national curriculum, but has many videos relevant to Australian schools.

https://www.teachingchannel.org/
## Numeracy resources

<table>
<thead>
<tr>
<th>Scootle</th>
<th>Scootle.edu.au gives teachers access to many thousands of digital curriculum resources they can use to inform their own planning and support their teaching. The resources include learning objects, images, videos, audio, assessment resources, teacher resources and collections organised around common topics or themes. The resources are aligned to the endorsed areas of the Australian Curriculum. Logging in: Teachers in Tasmanian government schools log in using their DoE username and password.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachertube</td>
<td>A collection of videos, audios, photos, blogs and documents for teachers, parents and students. <a href="http://www.teachertube.com">www.teachertube.com</a></td>
</tr>
<tr>
<td>Top Drawer Teachers</td>
<td><a href="http://topdrawer.aamt.edu.au/">http://topdrawer.aamt.edu.au/</a> Top Drawer Teachers has a wealth of ideas on key mathematical ideas including advice on assessment, planning and student tasks. Developed by the Australian Association of Mathematics Teachers.</td>
</tr>
<tr>
<td>NZ Maths</td>
<td>New Zealand Maths site – lots of resources and ideas to support teaching mathematics for numeracy <a href="http://nzmaths.co.nz/">http://nzmaths.co.nz/</a></td>
</tr>
</tbody>
</table>